MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

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Maisammaguda, Dhulapally (Post Via. Kompally), Secunderabad – 500100, Telangana State, India

DEPARTMENT OF ELECTRICAL & ELECTRONICS ENGINEERING

DIGITAL NOTES for POWER SYSTEMS - II

For

B. Tech(EEE) – II YEAR – II SEMESTER

Prepared by
Mr. R.SAI KIRAN, Assistant Professor/ EEE

MALLA REDDY COLLEGE OF ENGINEERING AND TECHNOLOGY

II YEAR B. Tech EEE– II SEM **L/T/P/C**

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(R22A0206) POWER SYSTEM - II

Pre requisite: Power system-1 & Electromagnetic Fields

COURSE OBJECTIVES:

- To know about AC distribution systems
- To understand the concept of voltage control & PF improvement.
- To understand and develop Y bus matrices
- To understand and develop Z bus matrices
- To understand the concepts load flow studies.

UNIT-I: A.C. DISTRIBUTION: Introduction, AC distribution, Single phase, 3-phase, 3 phase 4 wire system, bus bar arrangement. Voltage Drop Calculations (Numerical Problems) in A.C. Distributors for the following cases: Power Factors referred to receiving end voltage and with respect to respective load voltages.

UNIT-II: VOLTAGE CONTROL & POWER FACTOR IMPROVEMENT: Introduction – methods of voltage control, shunt and series capacitors / Inductors, tap changing transformers, synchronous phase modifiers, power factor improvement methods.

UNIT III:

POWER SYSTEM NETWORK MATRICES: Bus Incidence Matrix, Y-bus formation by Direct and Singular Transformation Methods, Numerical Problems.

UNIT IV:

FORMATION OF Z-BUS: Partial network, Algorithm for the Modification of Z Bus Matrix for addition element for the following cases: Addition of element from a new bus to reference, Addition of element from a new bus to an old bus, Addition of element between an old bus to reference and addition of element between two old buses

UNIT–V

LOAD FLOW STUDIES: Derivation of Static load flow equations. Load Flow Solutions Using Gauss Seidel Method & Newton Raphson Method (Polar coordinates only): Acceleration Factor, Load flow solution with and without P-V buses, Algorithm and Flowchart. Numerical Load flow Solution for Simple Power Systems (Max. 3-Buses): Determination of Bus Voltages, Injected Active and Reactive Powers (Sample One Iteration only) and finding Line Flows/Losses for the given Bus Voltages.

TEXT BOOKS: 1. C.L. Wadhwa, Electrical Power Systems, 3rd Edn, New Age International Publishing Co., 2001.

2. D.P.Kothari and I.J.Nagrath, Modern Power System Analysis, 4th Edn, Tata McGraw Hill Education Private Limited2011.

REFERENCE BOOKS:

1. D. P. Kothari: Modern Power System Analysis-Tata McGraw Hill Pub. Co. 2003

2. Hadi Scadat: Power System Analysis – Tata McGraw Hill Pub. Co.2002

3. W.D. Stevenson: Elements of Power system Analysis – McGraw Hill International Student Edition.

COURSE OUTCOMES:

At the end of the course the student will be able to:

- Analyze the operations of AC Distribution systems.
- Analyze voltage Control and Power factor improvement.
- Evaluate the admittance matrix of a given power systems.
- Evaluate the impedance matrix of a given power systems.
- Understand the concept of load flow studies in power system.

UNIT-I

A.C. DISTRIBUTION

Introduction, Types of AC distribution Systems, Voltage Drop Calculations (Numerical Problems) in A.C. Distributors for the following cases: Power Factors referred to receiving end voltage and with respect to respective load voltages.

INTRODUCTION:

Now-a-days, electrical energy is generated, transmitted and distributed in the form of alternating current as an economical proposition. The electrical energy produced at the power station is transmitted at very high voltages by 3-phase, 3- wire system to step-down sub-stations for distribution. The distribution system consists of two parts viz. primary distribution and secondary distribution. The primary distribution circuit is 3- phase, 3-wire and operates at voltages (3·3 or 6·6 or 11kV) somewhat higher than general utilisation levels. It delivers power to the secondary distribution circuit through distribution transformers situated near consumers' localities. Each distribution transformer steps down the voltage to 400 V and power is distributed to ultimate consumers' by 400/230 V, 3-phase, 4-wire system.

A.C. DISTRIBUTION:

Now-a-days electrical energy is generated, transmitted and distributed in the form of alternating current. One important reason for the widespread use of alternating current in preference to direct current is the fact that alternating voltage can be conveniently changed in magnitude by means of a transformer. Transformer has made it possible to transmit A.C. power at high voltage and utilise it at a safe potential. High transmission and distribution voltages have greatly reduced the current in the conductors and the resulting line losses.

There is no definite line between transmission and distribution according to voltage or bulk capacity. However, in general, the A.C. distribution system is the electrical system between the step down substation fed by the transmission system and the consumer's meters.

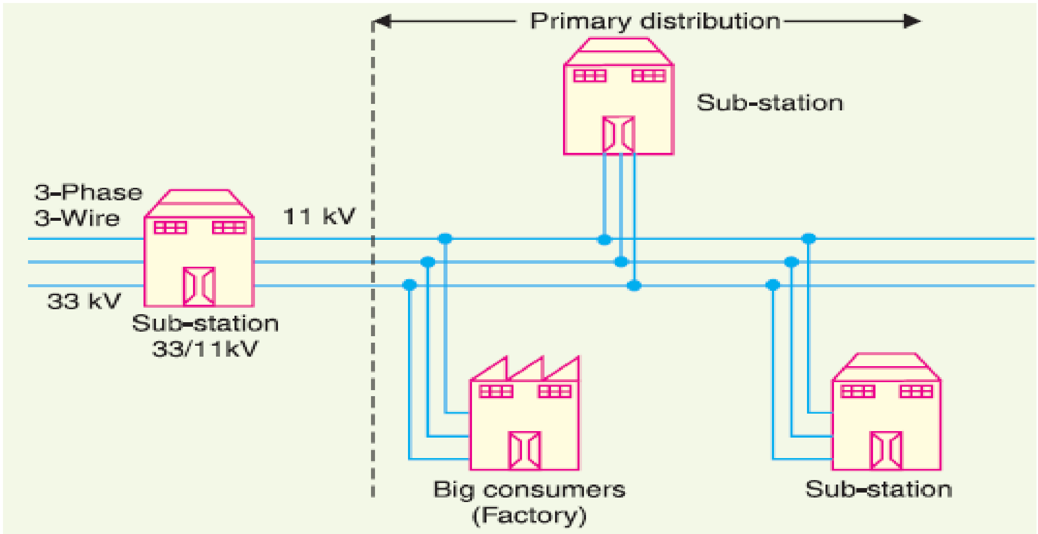
The A.C. distribution system is classified into

(i) Primary distribution system and

(ii) Secondary distribution system.

(i) Primary distribution system:

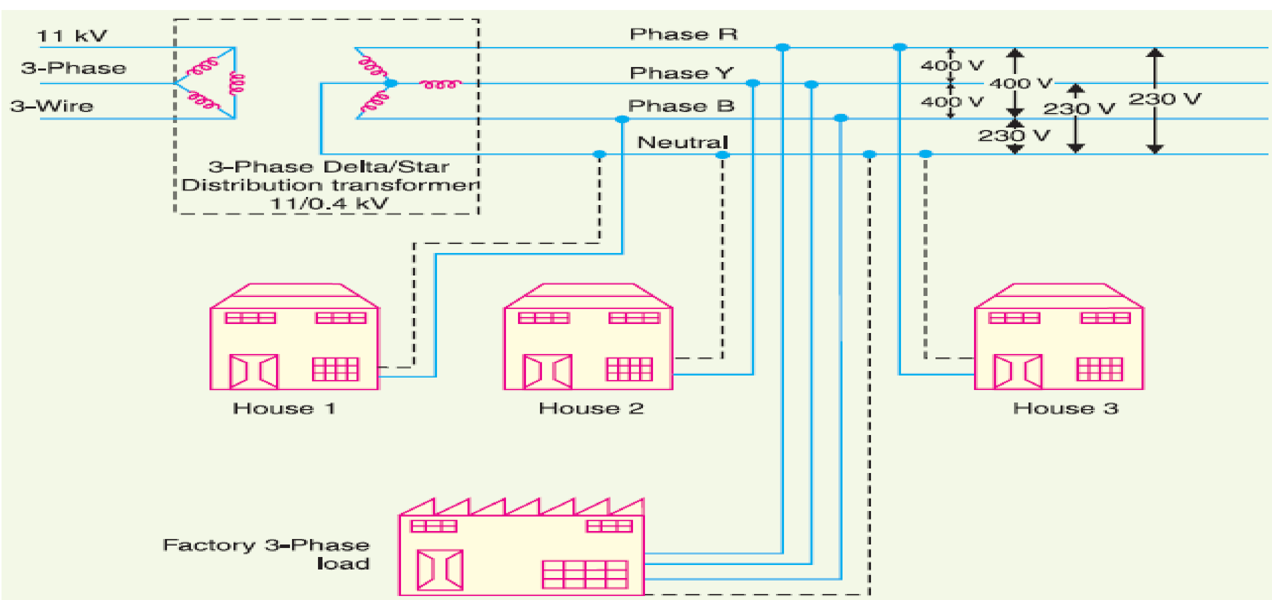
It is the part of A.C. distribution system which operates at voltages somewhat higher than general utilisation and handles large blocks of electrical energy than the average low-voltage consumer uses. The voltage used for primary distribution depends upon the amount of power to be conveyed and the distance of the substation required to be fed. The most commonly used primary distribution voltages are 11 kV, 6·6 kV and 3·3kV. Due to economic considerations, primary distribution is carried out by 3- phase, 3-wire system.



The above fig. shows a typical primary distribution system. Electric power from the generating station is transmitted at high voltage to the substation located in or near the city. At this substation, voltage is stepped down to 11 kV with the help of step-down transformer. Power is supplied to various substations for distribution or to big consumers at this voltage. This forms the high voltage distribution or primary distribution.

(ii) Secondary distribution system:

It is that part of A.C. distribution system which includes the range of voltages at which the ultimate consumer utilises the electrical energy delivered to him. The secondary distribution employs 400/230V, 3- phase, 4-wire system. The below fig. shows a typical secondary distribution system.



The primary distribution circuit delivers power to various substations, called distribution substations. The substations are situated near the consumers' localities and contain step down transformers. At each distribution substation, the voltage is stepped down to 400 V and power is delivered by 3-phase, 4-wire A.C. system. The voltage between any two phases is 400 V and between any phase and neutral is 230 V. The single phase domestic loads are connected between any one phase and the neutral, whereas 3-phase 400 V motor loads are connected across 3- phase lines directly.

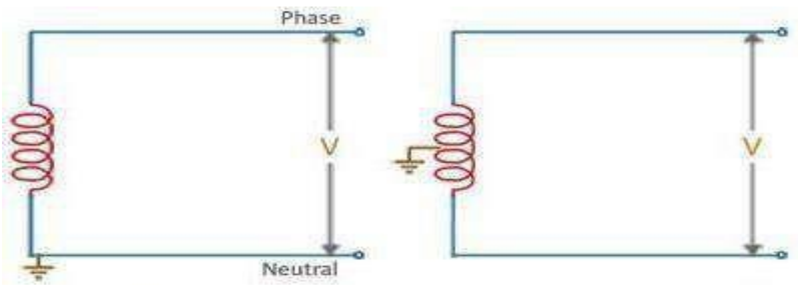
TYPES OF AC POWERDISTRIBUTIONSYSTEMS:

According to phases and wires involved, an AC distribution system can be classified as

1. Single phase, 2-wire system
2. Single phase, 3-wire system
3. Two phase, 3-wire system
4. Two phase, 4-wire system
5. Three phase, 3-wire system
6. Three phase, 4-wire system

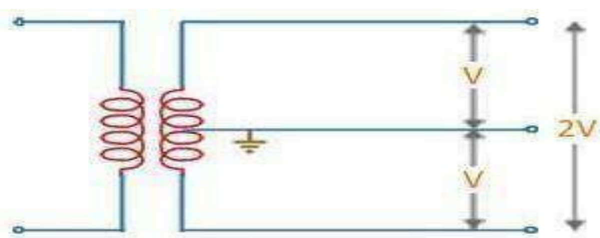
1. Single Phase, 2-Wire system:

This system may be used for very short distances. The following figure shows a single phase two wire system with one of the two wires earthed and mid-point of the phase winding is earthed.



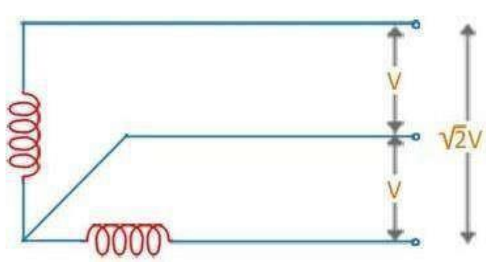
2. Single Phase, 3-Wire System:

This system is identical in principle with 3-wire dc distribution system. The neutral wire is center-tapped from the secondary winding of the transformer and earthed. This system is also called as split-phase electricity distribution system**.** It is commonly used in North America for residential supply.



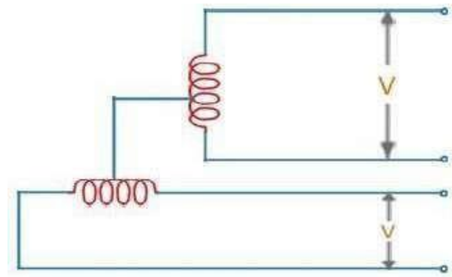
3. Two Phase, 3-Wire System:

In this system, the neutral wire is taken from the junction of two phase windings whose voltages are in quadrature with each other. The voltage between neutral wire and either of the outer phase wires is V. whereas; the voltage between outer phase wires is √2V. As compared to a two-phase 4-wire system, this system suffers from voltage imbalance due to unsymmetrical voltage in the neutral.



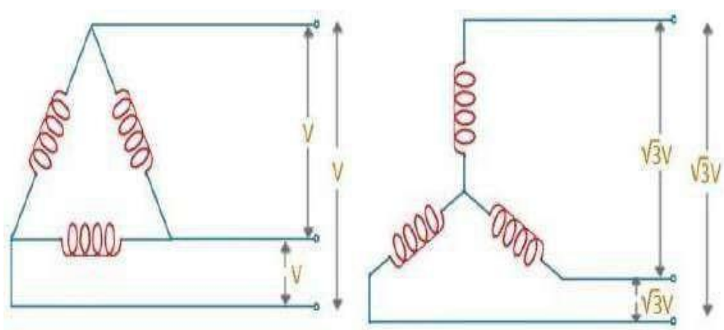
4. Two Phase, 4-Wire System

In this system, 4 wires are taken from two phase windings whose voltages are in quadrature with each other. Mid-point of both phase windings is connected together. If the voltage between the two wires of a same phase is V, then the voltage between two wires of different phase would be 0.707V.



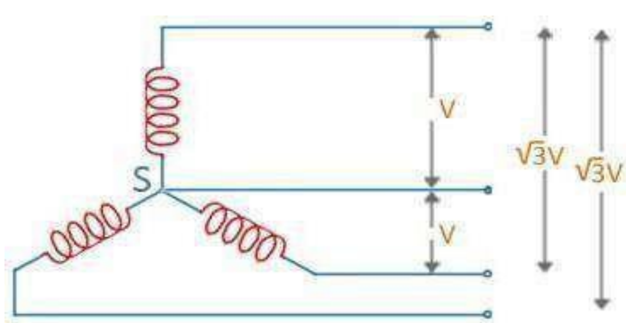
5. Three Phase, 3-Wire Distribution System

Three phase systems are very widely used for AC power distribution. The three phases may be delta connected or star connected with star point usually grounded. The voltage between two phases or lines for delta connection is V, where V is the voltage across a phase winding. For star connection, the voltage between two phases is √3V.



6. Three Phase, 4-Wire Distribution System

This system uses star connected phase windings and the fourth wire or neutral wire is taken from the star point. If the voltage of each winding is V, then the line-to-line voltage (line voltage) is √3V and the line-to- neutral voltage (phase voltage) is V. This type of distribution system is widely used in India and many other countries. In these countries, standard phase voltage is 230 volts and line voltage is √3x230 = 400 volts. Single phase residential loads, single phase motors which run on 230 volts etc. are connected between any one phase and the neutral. Three phase loads like three-phase induction motors are put across all the three phases and the neutral.



A.C. DISTRIBUTION CALCULATIONS:

A.C. distribution calculations differ from those of d.c. distribution in the following respects:

- In case of d.c. system, the voltage drop is due to resistance alone. However, in a.c. system, the voltage drops are due to the combined effects of resistance, inductance and capacitance.
- In a d.c. system, additions and subtractions of currents or voltages are done arithmetically but in case of a.c. system, these operations are done vectorially.
- In an a.c. system, power factor (p.f.) has to be taken into account. Loads tapped off form the distributor are generally at different power factors. There are two ways of referring power factor *viz*
  a. It may be referred to supply or receiving end voltage which is regarded as the reference vector.
  b. It may be referred to the voltage at the load point itself.

There are several ways of solving a.c. distribution problems. However, symbolic notation method has been found to be most convenient for this purpose. In this method, voltages, currents and impedances are expressed in complex notation and the calculations are made exactly as in d.c. distribution.

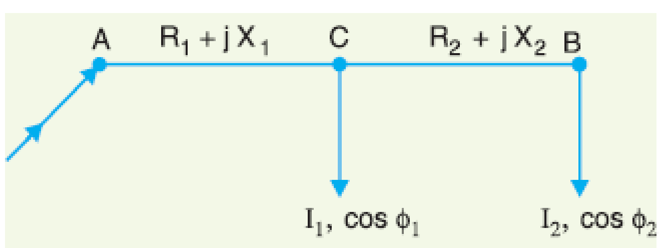
VOLTAGE DROP CALCULATIONS IN A.C. DISTRIBUTORS:

In a.c. distribution calculations, power factors of various load currents have to be considered since currents in different sections of the distributor will be the vector sum of load currents and not the arithmetic sum. The power factors of load currents may be given

1. With respect to receiving or sending end voltage
2. With respect to load voltage itself.

1. Power factors referred to receiving end voltage:

Consider an a.c. distributor *AB* with concentrated loads of I_1 and I_2 tapped off at points *C* and *B* as shown in below fig. Taking the receiving end voltage V_B as the reference vector, let lagging power factors at C and B is $\cos\varphi_1$ and $\cos\varphi_2$ with respect to V_B. Let R_1, X_1 and R_2, X_2 be the resistance and reactance of sections AC and CB of the distributor.



Impedance of section AC, $\vec{Z}_{AC} = R_1 + j X_1$

Impedance of section CB, $\vec{Z}_{CB} = R_2 + j X_2$

Load current at point C, $\vec{I}_1 = I_1 (\cos\varphi_1 - j \sin\varphi_1)$

Load current at point B, $\vec{I}_2 = I_2 (\cos\varphi_2 - j \sin\varphi_2)$

Current in section CB, $\vec{I}_{CB} = \vec{I}_2 = I_2 (\cos\varphi_2 - j \sin\varphi_2)$

Current in section AC, $\vec{I}_{AC} = \vec{I}_1 + \vec{I}_2$

$= I_1 (\cos\varphi_1 - j \sin\varphi_1) + I_2 (\cos\varphi_2 - j \sin\varphi_2)$

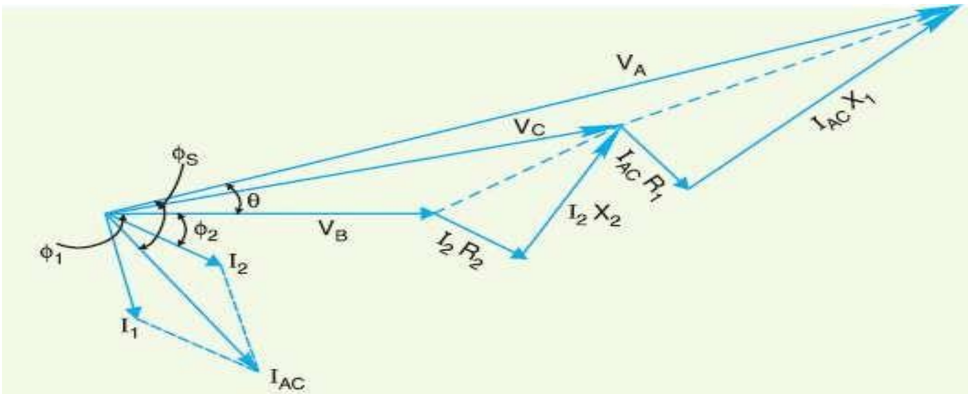
Voltage drop in section CB, $\vec{V}_{CB} = \vec{I}_{CB} \vec{Z}_{CB} = I_2 (\cos\varphi_2 - j \sin\varphi_2) (R_2 + j X_2)$

Voltage drop in section AC, $\vec{V}_{AC} = \vec{I}_{AC} \vec{Z}_{AC} = (\vec{I}_1 + \vec{I}_2) (R_1 + j X_1)$

$= [I_1 (\cos\varphi_1 - j \sin\varphi_1) + I_2 (\cos\varphi_2 - j \sin\varphi_2)] (R_1 + j X_1)$

Sending end voltage, $\vec{V}_A = \vec{V}_B + \vec{V}_{CB} + \vec{V}_{AC}$

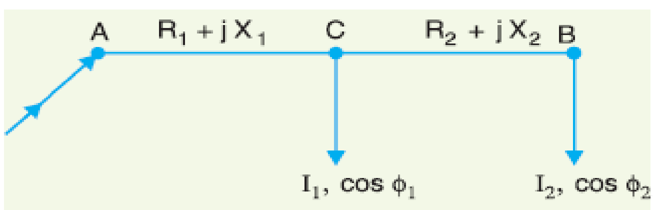
Sending end current, $\vec{I}_A = \vec{I}_1 + \vec{I}_2$



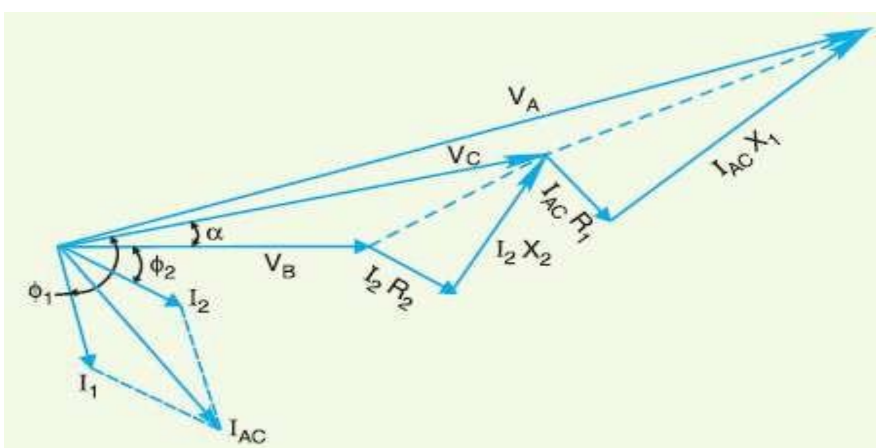
The vector diagram of the a.c. distributor under these conditions is shown in below fig. Here, the receiving end voltage V_B is taken as the reference vector. As power factors of loads are given with respect to V_B, therefore, I_1 and I_2 lag behind V_B by φ_1 and φ_2 respectively.

2. Power factors referred to respective load voltages:

The power factors of loads in the below fig. are referred to their respective load voltages.



Then φ_1 is the phase angle between V_C and I_1 and φ_2 is the phase angle between V_B and I_2. The vector diagram under these conditions is shown in below fig.



Voltage drop in section $CB = \vec{I_2}\,\vec{Z_{CB}} = I_2(\cos\varphi_2 - j\sin\varphi_2)(R_2 + jX_2)$

Voltage at point $C = V_B$ + Drop in section CB $= V_C \angle \alpha$ (say)

Now $\vec{I_1} = I_1 \angle -\varphi_1$ *w.r.t.* voltage V_C

$\therefore \vec{I_1} = I_1 \angle -(\varphi_1 - \alpha)$ *w.r.t.* voltage V_B

i.e. $\vec{I_1} = I_1[\cos(\varphi_1 - \alpha) - j\sin(\varphi_1 - \alpha)]$.

Now $\vec{I_{AC}} = \vec{I_1} + \vec{I_2}$

$= I_1[\cos(\varphi_1 - \alpha) - j\sin(\varphi_1 - \alpha)] + I_2(\cos\varphi_2 - j\sin\varphi_2)$ Voltage drop in section AC =

$\vec{I_{AC}}\,\vec{Z_{AC}}$

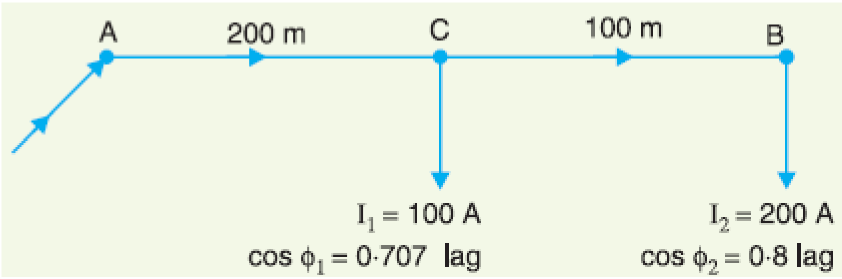
$\therefore$ Voltage at point A $= V_B$ + Drop in CB + Drop in AC

PROBLEMS:

1. A single phase a.c. distributor AB 300 metres long is fed from end A and is loaded as 100 A at 0·707 p.f. lagging 200 m from point A and 200 A at 0·8 p.f. lagging 300 m from point A. The load resistance and reactance of the distributor is 0·2 Ω and 0·1 Ω per kilometre. Calculate the total voltage drop in the distributor. The load power factors refer to the voltage at the far end.

The below fig shows the single line diagram of the distributor.

Impedance of distributor/km = $(0{\cdot}2 + j\,0{\cdot}1)\ \Omega$



Impedance of section AC, $\vec{Z_{AC}} = (0{\cdot}2 + j\,0{\cdot}1) \times 200/1000 = (0{\cdot}04 + j\,0{\cdot}02)\ \Omega$

Impedance of section CB, $\vec{Z_{CB}} = (0{\cdot}2 + j\,0{\cdot}1) \times 100/1000 = (0{\cdot}02 + j\,0{\cdot}01)\ \Omega$

Taking voltage at the far end B as the reference vector, we have,

Load current at point B, $\vec{I_2} = I_2(\cos\varphi_2 - j\sin\varphi_2) = 200(0{\cdot}8 - j\,0{\cdot}6) = (160 - j\,120)$ A

Load current at point C, $\vec{I_1} = I_1(\cos\varphi_1 - j\sin\varphi_1) = 100(0{\cdot}707 - j\,0{\cdot}707) = (70{\cdot}7 - j\,70{\cdot}7)$ A

Current in section CB, $\vec{I_{CB}} = I_2 = (160 - j\,120)$ A

Current in section AC, $\vec{I_{AC}} = I_1 + I_2 = (70{\cdot}7 - j\,70{\cdot}7) + (160 - j\,120) = (230{\cdot}7 - j\,190{\cdot}7)$ A

Voltage drop in section CB, $\vec{V_{CB}} = I_{CB}Z_{CB} = (160 - j\,120)(0{\cdot}02 + j\,0{\cdot}01) = (4{\cdot}4 - j\,0{\cdot}8)$ volts

Voltage drop in section AC, $\vec{V_{AC}} = I_{AC}Z_{AC} = (230{\cdot}7 - j\,190{\cdot}7)(0{\cdot}04 + j\,0{\cdot}02) = (13{\cdot}04 - j\,3{\cdot}01)$ volts

Voltage drop in the distributor $= \vec{V_{AC}} + \vec{V_{CB}} = (13{\cdot}04 - j\,3{\cdot}01) + (4{\cdot}4 - j\,0{\cdot}8) = (17{\cdot}44 - j\,3{\cdot}81)$ volts

Magnitude of drop $= \sqrt{17.44^2 + 3.81^2} = 17.85$v

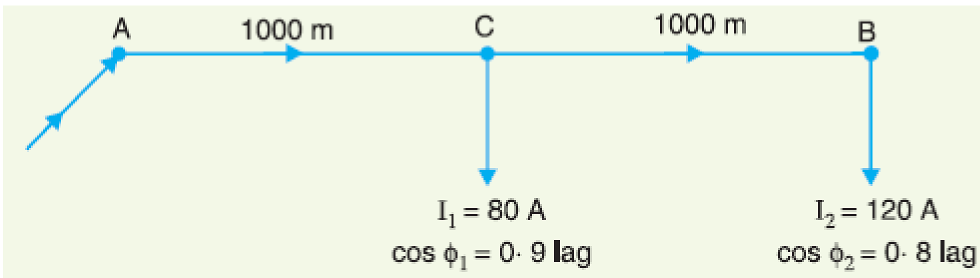
2. A single phase distributor 2 kilometres long supplies a load of 120 A at 0·8 p.f. lagging at its far end and a load of 80 A at 0·9 p.f. lagging at its mid-point. Both power factors are referred to the voltage at the far end. The resistance and reactance per km (go and return) are 0·05 Ω and 0·1 Ω respectively. If the voltage at the far end is maintained at 230 V, calculate, (i) voltage at the sending end (ii) phase angle between voltages at the two ends.

The below fig shows the distributor *AB* with *C* as the mid-point

Impedance of distributor/km = (0·05 + *j* 0·1) Ω

Impedance of section *AC*, $\vec{Z}_{AC}$ = (0·05 + *j* 0·1) × 1000/1000 = (0·05 + *j* 0·1) Ω

Impedance of section *CB*, $\vec{Z}_{CB}$ = (0·05 + *j* 0·1) × 1000/1000 = (0·05 + *j* 0·1) Ω



Let the voltage V_B at point *B* be taken as the reference vector.

Then, $\vec{V}_B$ = 230 + *j* 0

i. Load current at point *B*, $\vec{I}_2$ = 120 (0·8 − *j* 0·6) = 96 − *j* 72

Load current at point *C*, $\vec{I}_1$ = 80 (0·9 − *j* 0·436) = 72 − *j* 34·88

Current in section *CB*, $\vec{I}_{CB} = \vec{I}_2$ = 96 − *j* 72

Current in section *AC*, $\vec{I}_{AC} = \vec{I}_1 + \vec{I}_2$ = (72 − *j* 34·88) + (96 − *j* 72)

= 168 − *j* 106·88

Drop in section *CB*, $\vec{V}_{CB} = \vec{I}_{CB}\,\vec{Z}_{CB}$ = (96 − *j* 72) (0·05 + *j* 0·1)

= 12 + *j* 6

Drop in section *AC*, $\vec{V}_{AC} = \vec{I}_{AC}\,\vec{Z}_{AC}$ = (168 − *j* 106·88) (0·05 + *j* 0·1)

= 19·08 + *j* 11·45

∴ Sending end voltage, $\vec{V}_A = \vec{V}_B + \vec{V}_{CB} + \vec{V}_{AC}$

= (230 + *j* 0) + (12 + *j* 6) + (19.08 + *j* 11.45)

= 261.08 + *j* 17.45

Its magnitude is = $\sqrt{261.08^2 + 17.45^2}$ = 261.67 V

ii. The phase difference θ between *VA* and *VB* is given by:

$$\tan\theta = \frac{17.45}{261.08} = 0.0668$$

$$\theta = \tan^{-1} 0{\cdot}0668 = 3{\cdot}82^\circ$$

3. A single phase distributor one km long has resistance and reactance per conductor of 0·1 Ω and 0·15 Ω respectively. At the far end, the voltage V_B = 200 V and the current is 100 A at a p.f. of 0·8 lagging. At the mid-point M of the distributor, a current of 100 A is tapped at a p.f. of 0·6 lagging with reference to the voltage V_M at the mid-point. Calculate (i) Voltage at mid-point (ii) Sending end voltage V_A (iii) Phase angle between V_A and V_B.

The below fig shows the single line diagram of the distributor *AB* with *M* as the mid-point.

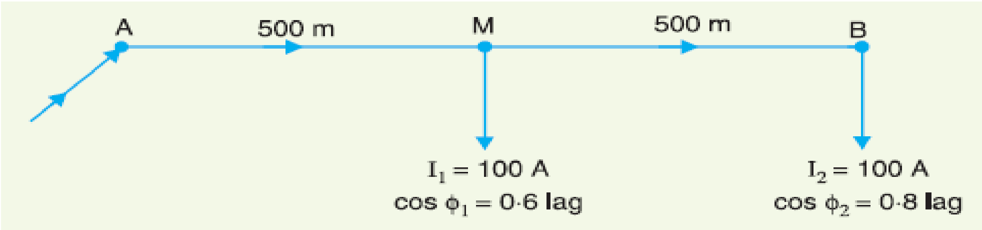
Total impedance of distributor = 2(0·1 + *j* 0·15) = (0·2 + *j* 0·3) Ω

Impedance of section *AM*, $\vec{Z}_{AM}$ = (0·1 + *j* 0·15) Ω

Impedance of section *MB*, $\vec{Z}_{MB}$ = (0·1 + *j* 0·15) Ω

Let the voltage V_B at point *B* be taken as the reference vector.

Then, $\vec{V}_B = 200 + j\,0$



(i) Load current at point *B*, $\vec{I}_2 = 100\,(0{\cdot}8 - j\,0{\cdot}6) = 80 - j\,60$

Current in section *MB*, $\vec{I}_{MB} = \vec{I}_2 = 80 - j\,60$

Drop in section *MB*, $\vec{V}_{MB} = \vec{I}_{MB}\,\vec{Z}_{MB}$

$= (80 - j\,60)\,(0{\cdot}1 + j\,0{\cdot}15) = 17 + j\,6$

∴ Voltage at point *M*, $\vec{V}_M = \vec{V}_B + \vec{V}_{MB} = (200 + j\,0) + (17 + j\,6)$

$= 217 + j\,6$

Its magnitude is = $\sqrt{217^2 + 6^2}$ = 217.1V

Phase angle between V_M and V_B, $\alpha = \tan^{-1} 6/217 = \tan^{-1} 0{\cdot}0276 = 1{\cdot}58^o$

(ii) The load current I_1 has a lagging p.f. of 0·6 *w.r.t.* V_M. It lags behind V_M by an angle

$\varphi_1 = \cos^{-1} 0{\cdot}6 = 53{\cdot}13^o$

∴ Phase angle between I_1 and V_B, $\varphi'_1 = \varphi_1 - \alpha = 53{\cdot}13^o - 1{\cdot}58 = 51{\cdot}55^o$

Load current at *M*, $\vec{I}_1 = I_1\,(\cos\varphi_1' - j\sin\varphi_1') = 100\,(\cos 51{\cdot}55^\circ - j\sin 51{\cdot}55^\circ)$

$= 62{\cdot}2 - j\,78{\cdot}3$

Current in section *AM*, $\vec{I}_{AM} = \vec{I}_1 + \vec{I}_2 = (62{\cdot}2 - j\,78{\cdot}3) + (80 - j\,60)$

$= 142{\cdot}2 - j\,138{\cdot}3$

Drop in section *AM*, $\vec{V}_{AM} = \vec{I}_{AM}\,\vec{Z}_{AM}$

$= (142{\cdot}2 - j\,138{\cdot}3)\,(0{\cdot}1 + j\,0{\cdot}15)$

$= 34{\cdot}96 + j\,7{\cdot}5$

Sending end voltage, $\vec{V}_A = \vec{V}_M + \vec{V}_{AM} = (217 + j\,6) + (34{\cdot}96 + j\,7{\cdot}5)$

$$= 251{\cdot}96 + j\,13{\cdot}5$$

Its magnitude is $= \sqrt{251.96^2 + 13.5^2} = 252.32$ V

(iii) The phase difference θ between V_A and V_B is given by

$$\tan\theta = 13{\cdot}5/251{\cdot}96 = 0{\cdot}05358$$

$$\therefore \theta = \tan^{-1} 0{\cdot}05358 =$$

$3{\cdot}07^o$ Hence supply voltage is 252·32 V and leads V_B by $3{\cdot}07^\circ$.

UNIT-II

Voltage Control

Introduction

In a modern power system, electrical energyfrom the generating station is delivered to the ultimate consumers through a network of transmission and distribution. For satisfactory op-eration of motors, lamps and other loads, it is de-sirable that consumers are supplied with substan-tially constant voltage.

Too wide variations ofvoltage may cause erratic operation or even mal-functioning of consumers' appliances. To safe-guard the interest of the consumers, the govern- ment has enacted a law in this regard. The statu-tory limit of voltage variation is ± 6% of declared voltage at consumers' terminals.

The principal cause of voltage variation at consumer's premises is the change in load on the supply system. When the load on the system in-creases, the voltage at the consumer's terminals falls due to the increased voltage drop in ***(i)*** al- ternator synchronous impedance ***(ii)*** transmission line ***(iii)*** transformer impedance ***(iv)*** feeders and **(v)** distributors. The reverse would happen should the load on the system decrease. These voltage variations are undesirable and must be kept within the prescribed limits (*i.e.* ± 6% of the declared voltage). This is achieved by installing voltageregulating equipment at suitable places in the power system. The purpose of this chapter is to deal with important voltage control equipment and its increasing utility in this fast developing power system.

15.1 Importance of Voltage Control

When the load on the supply system changes, the voltage at the consumer's terminals also changes. The variations of voltage at the consumer's terminals are undesirable and must be kept within pre- scribed limits for the following reasons :

(i) In case of lighting load, the lamp characteristics are very sensitive to changes of voltage. For instance, if the supply voltage to an incandescent lamp decreases by 6% of rated value, then illuminating power may decrease by 20%. On the other hand, if the supply voltage is 6% above the rated value, the life of the lamp may be reduced by 50% due to rapid deterioration of the filament.

(ii) In case of power load consisting of induction motors, the voltage variations may cause erratic operation. If the supply voltage is above the normal, the motor may operate with a saturated magnetic circuit, with consequent large magnetising current, heating and low power factor. On the other hand, if the voltage is too low, it will reduce the starting torque of the motor considerably.

(iii) Too wide variations of voltage cause excessive heating of distribution transformers. This may reduce their ratings to a considerable extent.

It is clear from the above discussion that voltage variations in a power system must be kept tominimum level in order to deliver good service to the consumers. With the trend towards larger and larger interconnected system, it has become necessary to employ appropriate methods of voltage control.

15.2 Location of Voltage Control Equipment

In a modern power system, there are several elements between the generating station and the consum-ers. The voltage control equipment is used at more than one point in the system for two reasons. Firstly, the power network is very extensive and there is a considerable voltage drop in transmissionand distribution systems. Secondly, the various circuits of the power system have dissimilar load characteristics. For these reasons , it is necessary to provide individual means of voltage control foreach circuit or group of circuits. In practice, voltage control equipment is used at :

(i) generating stations
(ii) transformer stations
(iii) the feeders if the drop exceeds the permissible limits

15.3 Methods of Voltage Control

There are several methods of voltage control. In each method, the system voltage is changed in accordance with the load to obtain a fairly constant voltage at the consumer's end of the system. Thefollowing are the methods of voltage control in an *a.c. power system:

(i) By excitation control
(ii) By using tap changing transformers
(iii) Auto-transformer tap changing
(iv) Booster transformers
(v) Induction regulators
(vi) By synchronous condenser

Method (*i*) is used at the generating station only whereas methods (*ii*) to (*v*) can be used for

* Since the modern power system is a.c., voltage control for this system will be discussed. However, for a d.c. system, voltage control can be effected by (*i*) overcompounded generators and (*ii*) boosters.

transmission as well as primary distribution systems. However, methods (*vi*) is reserved for the voltage control of a transmission line. We shall discuss each method separately in the next sections.

15.4 Excitation Control

When the load on the supply system changes, the terminal voltage of the alternator also varies due to the changed voltage drop in the synchronous reactance of the armature. The voltage of the alternatorcan be kept constant by changing the *field current of the alternator in accordance with the load. This is known as *excitation control* method. The excitation of alternator can be controlled by the use of automatic or hand operated regulator acting in the field circuit of the alternator. The first method ispreferred in modern practice. There are two main types of automatic voltage regulators *viz*.

(i) Tirril Regulator
(ii) Brown-Boveri Regulator

These regulators are based on the "overshooting the mark †principle" to enable them to respondquickly to the rapid fluctuations of load. When the load on the alternator increases, the regulator produces an increase in excitation more than is ultimately necessary. Before the voltage has the time to increase to the value corresponding to the increased excitation, the regulator reduces the excitationto the proper value.

15.5 Tirril Regulator

In this type of regulator, a fixed resistance is cut in and cut out of the exciter field circuit of the alternator. This is achieved by rapidly opening and closing a shunt circuit across the exciter rheostat.For this reason, it is also known as vibrating type voltage regulator.

Construction. Fig. 15.1 shows the essential parts of a Tirril voltage regulator. A rheostat R is provided in the exciter circuit and its value is set to give the required excitation. This rheostat is putin and out of the exciter circuit by the regulator, thus varying the exciter voltage to maintain the desired voltage of the alternator.

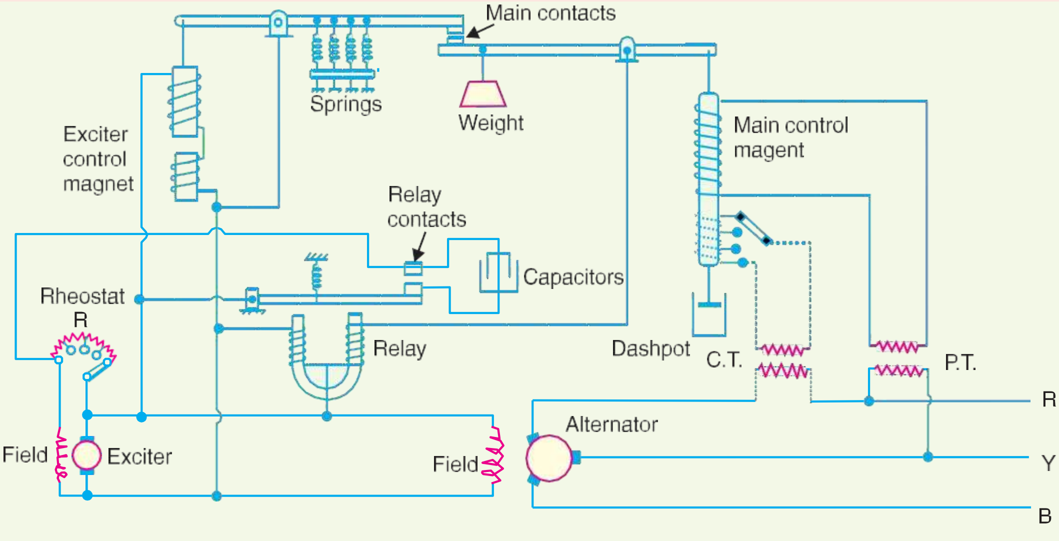


Fig. 15.1 Tirril Regulator

* As alternator has to be run at constant speed to obtain fixed frequency, therefore, induced e.m.f. of the alternator cannot be controlled by the adjustment of speed.

† The alternator has large inductance. If the exciter voltage is increased, the field current will take some time to reach the steady value. Therefore, response will not be quick. However, quick response is necessary to meet the rapid fluctuations of load. For this reason, this principle is used.

(i) *Main contact.* There are two levers at the top which carry the main contacts at the facing ends. The left-hand lever is controlled by the exciter magnet whereas the right hand lever is controlled by an a.c. magnet known as main control magnet.

(ii) *Exciter magnet.* This magnet is of the ordinary solenoid type and is connected across the exciter mains. Its exciting current is, therefore, proportional to the exciter voltage. The counter balancing force for the exciter magnet is provided by four coil springs.

(iii) *A. C. magnet.* It is also of solenoid type and is energised from a.c. bus-bars. It carries series as well as shunt excitation. This magnet is so adjusted that with normal load and voltage at the alternator, the pulls of the two coils are equal and opposite, thus keeping the right-hand lever in the horizontal position.

(iv) *Differential relay.* It essentially consists of a U-shaped relay magnet which operates the relay contacts. The relay magnet has two identical windings wound differentially on both the limbs. These windings are connected across the exciter mains–the left hand one permanently while the right hand one has its circuit completed only when the main contacts are closed. The relay contacts are arranged to shunt the exciter-field rheostat *R*. A capacitor is provided across the relay contacts to reduce the sparking at the time the relay contacts are opened.

Operation. The two control magnets (*i.e*. exciter magnet and a.c. magnet) are so adjusted thatwith normal load and voltage at the alternator, their pulls are equal, thus keeping the main contactsopen. In this position of main contacts, the relay magnet remains energised and pulls down the armature carrying one relay contact. Consequently, relay contacts remain open and the exciter fieldrheostat is in the field circuit.

When the load on the alternator increases, its terminal voltage tends to fall. This causes the series excitation to predominate and the a.c. magnet pulls down the right-hand lever to close the main contacts. Consequently, the relay magnet is *de-energised and releases the armature carrying the relay contact. The relay contacts are closed and the rheostat *R* in the field circuit is short circuited. This increases the exciter-voltage and hence the excitation of the alternator. The increased excitation causes the alternator voltage to rise quickly. At the same time, the excitation of the exciter magnet isincreased due to the increase in exciter voltage. Therefore, the left-hand lever is pulled down, open- ing the main contacts, energising the relay magnet and putting the rheostat *R* again in the field circuitbefore the alternator voltage has time to increase too far. The reverse would happen should the loadon the alternator decrease.

It is worthwhile to mention here that exciter voltage is controlled by the rapid opening and closing of the relay contacts. As the regulator is worked on the overshooting the mark principle, therefore, the terminal voltage does not remain absolutely constant but oscillates between the maxi- mum and minimum values. In fact, the regulator is so quick acting that voltage variations never exceed ± 1%.

15.6 Brown-Boveri Regulator

In this type of regulator, exciter field rheostat is varied continuously or in small steps instead of being first completely cut in and then completely cut out as in Tirril regulator. For this purpose, a regulating resistance is connected in series with the field circuit of the exciter. Fluctuations in the alternator voltage are detected by a control device which actuates a motor. The motor drives the regulating rheostat and cuts out or cuts in some resistance from the rheostat, thus changing the exciter and hence the alternator voltage.

Construction. Fig. 15.2 shows the schematic diagram of a Brown-Boveri voltage regulator. It

* Because the windings are wound differentially on the two limbs.

also works on the "overshooting the mark principle" and has thefollowing four important parts :

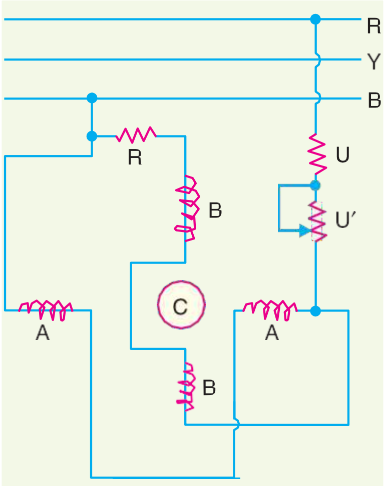


Fig. 15.2

(i) *Control system.* The control system is built on the principle of induction motor. It consists of two windings *A* and *B* on an annular core of laminated sheet steel. The winding *A* is excited from two of the generator terminals through resistances *U* and *U′* while a resistance *R* is inserted in the circuit of winding *B*. The ratio of resistance to reactance of the two windings are suitably adjusted so as to create a phase difference of currents in the two windings. Due to the phase difference of currents in the two windings, rotating magnetic field is set up. This produces electromagnetic torque on the thin aluminium drum *C* carried by steel spindle ; the latter being supported at both ends by jewel bearings. The torque on drum *C* varies with the terminal voltage of the alternator. The variable resistance *U′* can also vary the torque on the drum. If the resistance is increased, the torque is decreased and vice- versa. Therefore, the variable resistance *U′* provides a means by which the regulator may be set to operate at the desired voltage.

(ii) *Mechanical control torque.* The electric torque produced by the current in the split phase winding is opposed by a combination of two springs (main spring and auxiliary spring) which produce a constant mechanical torque irrespective of the position of the drum. Under steady deflected state, mechanical torque is equal and opposite to the electric torque.

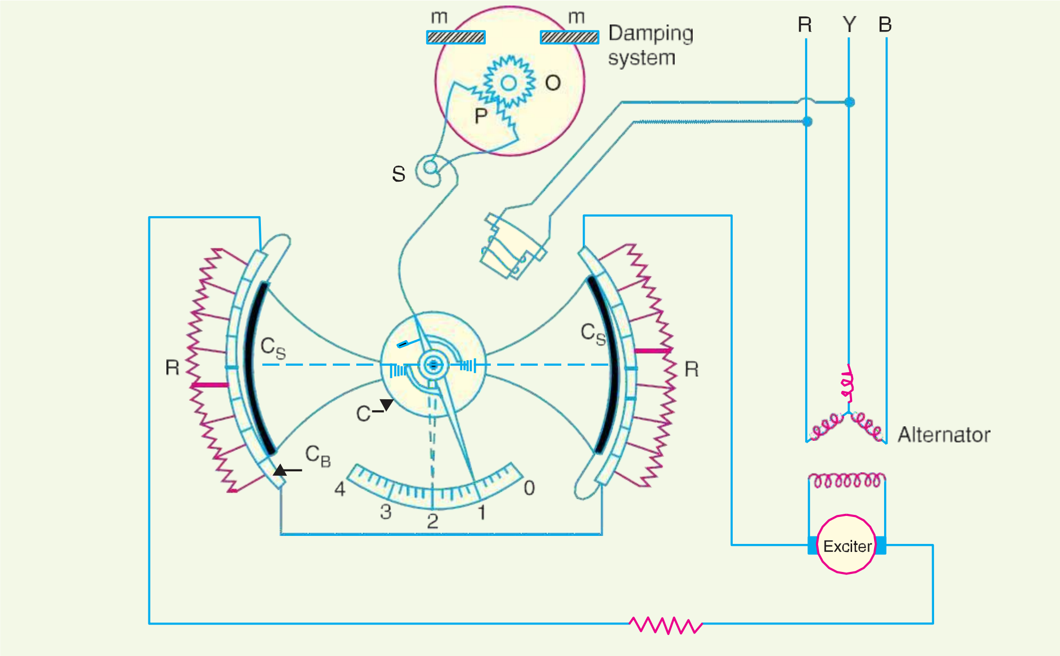


Fig. 15.3

(iii) *Operating system.* It consists of a field rheostat with contact device. The rheostat consists of a pair of resistance elements connected to the stationary contact blocks C_B. These two circuit of the exciter. On the inside surface of the contact blocks roll the contact sectors C_S.When the terminal voltage of the alternator changes, the electric torque acts on the drum. This causes the contact sectors to roll over the contact blocks, cutting in or cutting out rheostat resistance in the exciter field circuit.

(iv) *Damping torque.* The regulator is made stable by damping mechanism which consists of an aluminium disc *O* rotating between two permanent magnets *m*. The disc is geared to the rack of an aluminium sector *P* and is fastened to the aluminium drum *C* by means of a flexible spring *S* acting as the recall spring. If there is a change in the alternator voltage, the eddy currents induced in the disc *O* produce the necessary damping torque to resist quick response of the moving system.

Operation. Suppose that resistances *U* and *U′* are so adjusted that terminal voltage of the alter-nator is normal at position 1. In this position, the electrical torque is counterbalanced by the mechani- cal torque and the moving system is in equilibrium. It is assumed that electrical torque rotates the shaft in a clockwise direction.

Now imagine that the terminal voltage of the alternator rises due to decrease in load on the supply system. The increase in the alternator voltage will cause an increase in electrical torque which becomes greater than the mechanical torque. This causes the drum to rotate in clockwise direction, say to position 3. As a result, more resistance is inserted in the exciter circuit, thereby decreasing thefield current and hence the terminal voltage of the alternator. Meanwhile, the recall spring *S* is tightened and provides a counter torque forcing the contact roller back to position 2 which is the equilibrium position. The damping system prevents the oscillations of the system about the equilib-rium position.

15.7 Tap-Changing Transformers

The excitation control method is satisfactory only for relatively short lines. However, it is *not suitable for long lines as the voltage at the alternator terminals will have to be varied too much in order that the voltage at the far end of the line may be constant. Under such situations, the problem of voltage control can be solved by employing other methods. One important method is to use tap- changingtransformer and is commonly employed where main transformer is necessary. In this method, a number of tappings are provided on the secondary of the transformer. The voltage drop in the line is supplied by changing the secondary e.m.f. of the transformer through the adjustment of its numberof turns.

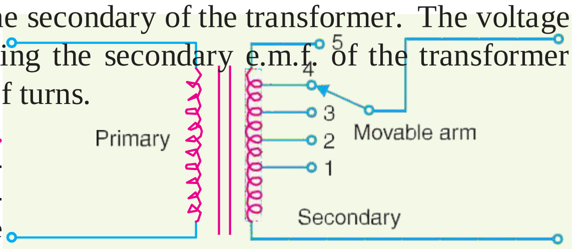


Fig. 15.4

(i) Off load tap-changing transformer. Fig. 15.4 shows the arrangement where a number of tappings have been provided on the secondary. As the position of the tap is varied, the effective number of secondary turns is varied and hence the output voltage of the secondary can be changed. Thus referring to Fig. 15.4, when the movable arm makes contact with stud 1, the secondary voltage is minimum and when with stud 5, it is maximum. During the period of light load, the voltage across the primary is not muchbelow the alternator voltage and the movable arm is placed on stud 1. When the load increases, the voltage across the primary drops, but the secondary voltage can be kept at the previous value by placing the movable arm on to a higher stud. Whenever a tapping is to be changed in this type of transformer, the load is kept off and hence the name off load tap-changing transformer.

* In a long line, difference in the receiving-end voltage between no load and full-load conditions is quite large.

The principal disadvantage of the circuit arrangement shown in Fig. 15.4 is that it cannot be used for tap-changing on load. Suppose for a moment that tapping is changed from position 1 to position2 when the transformer is supplying load. If contact with stud 1 is broken before contact with stud 2is made, there is break in the circuit and arcing results. On the other hand, if contact with stud 2 ismade before contact with stud 1 is broken, the coils connected between these two tappings are short- circuited and carry damaging heavy currents. For this reason, the above circuit arrangement cannotbe used for tap-changing on load.

(ii) On-load tap-changing transformer. In supply system, tap-changing has normally to be performed on load so that there is no interruption to supply. Fig. 15.5 shows diagrammatically one type of on-load tap-changing transformer. The secondary consists of two equal parallel windings which have similar tappings 1*a* ...... 5*a* and 1*b* .... 5*b*. In the normal working conditions, switches *a*, *b* and tappings with the same number remain closedand each secondary winding carries one-half of the total current. Referring to Fig. 15.5, the second- ary voltage will be maximum when switches *a*, *b*and 5*a*, 5*b* are closed. However, the secondary voltage will be minimum when switches *a*, *b* and1*a*, 1*b* are closed.

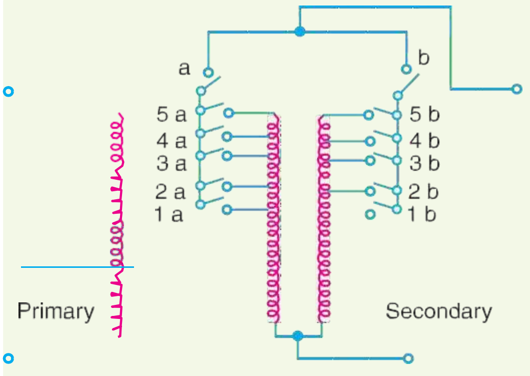


Fig. 15.5

Suppose that the transformer is working withtapping position at 4*a*, 4*b* and it is desired to alter its position to 5*a*, 5*b*. For this purpose, one of the switches *a* and *b*, say *a*, is opened. This takes thesecondary winding controlled by switch *a* out of the circuit. Now, the secondary winding controlledby switch *b* carries the total current which is twice its rated capacity. Then the tapping on the discon-nected winding is changed to 5*a* and switch *a* is closed. After this, switch *b* is opened to disconnectits winding, tapping position on this winding is changed to 5*b* and then switch *b* is closed. In this way, tapping position is changed without interrupting the supply. This method has the following disadvan- tages :

(i) During switching, the impedance of transformer is increased and there will be a voltage surge.

(ii) There are twice as many tappings as the voltage steps.

15.8 Auto-Transformer Tap-changing

Fig. 15.6 shows diagrammatically auto-transformer tap changing. Here, a mid-tapped auto-transformeror reactor is used. One of the lines is connected to its mid-tapping. One end, say *a* of this transformer is connected to a series of switches across the odd tappings and the other end *b* is connected to switchesacross even tappings. A short-circuiting switch *S* is connected across the auto-transformer and remainsin

the closed position under normal

operation. In the normal operation, there is *no inductive voltagedrop across the auto-transformer. Referring to Fig. 15.6, it is clear that with switch 5 closed, minimum

* In the normal operation, switch *S* remains closed so that half the total current flows through each half of the reactor. Since the currents in each half of the reactor are in opposition, no resultant flux is set up and consequently there is no inductive voltage drop across it.

secondary turns are in the circuit and hence the output voltage will be the lowest. On the other hand,the output voltage will be maximum when switch 1 is closed.

Suppose now it is desired to alter the tapping point from position 5 to position 4 in order to raisethe output voltage. For this purpose, short-circuiting switch *S* is opened, switch 4 is closed, then switch 5 is opened and finally short-circuiting switch is closed. In this way, tapping can be changedwithout interrupting the supply.

It is worthwhile to describe the electrical phenomenon occurring during the tap changing. When the short-circuiting switch is opened, the load current flows through one-half of the reactor coil so that there is a voltage drop across the reactor. When switch 4 is closed, the turns between points 4 and 5 are connected through the whole reactor winding. A circulating current flows through this local circuit but it is limited to a low value due to high reactance of the reactor.

15.9 Booster Transformer

Sometimes it is desired to control the voltage of a transmission line at a point far away from the maintransformer. This can be conveniently achieved by the use of a booster transformer as shown in Fig.
15.7. The secondary of the booster transformer is connected in series with the line whose voltage isto be controlled. The primary of this transformer is supplied from a regulating transformer *fitted with on-load tap-changing gear. The booster transformer is connected in such a way that its second-ary injects a voltage in phase with the line voltage.

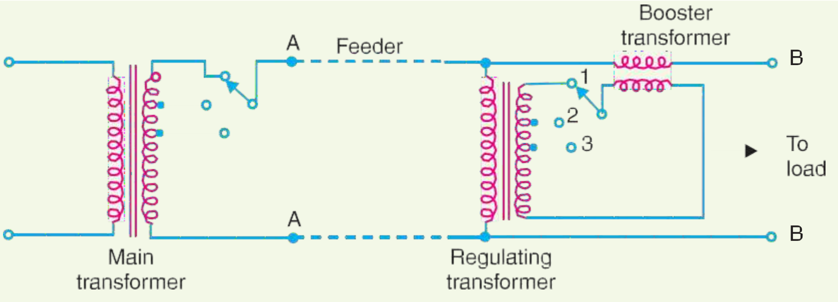
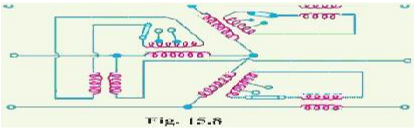


Fig. 15.7

The voltage at *AA* is maintained constant by tap-changing gear in the main transformer. However, there may be considerable volt-age drop between *AA* and *BB* due to fairly long feeder and tappingof loads. The voltage at *BB* is controlled by the use of regulating



transformer and booster transformer. By changing the tap- ping on the regulating transformer, the mag- nitude of the voltage injected into the line can be varied. Thispermits to keep the voltage at *BB* to the de- sired value. This method of voltage control has three disadvantages.Firstly, it is more expensive than the on-load tap-changing trans former. Secondly, it is less efficient owing to losses in the booster and thirdly more floor space isrequired. Fig. 15.8 shows a three-phase booster transformer.

15.10 The on-load tap-changing gear is omitted from the diagram for the sake of simplicity.

15.11 nduction Regulators

An induction regulator is essentially a constant voltage transformer, one winding of which can be moved *w.r.t.* the other, thereby obtaining a variable secondary voltage. The primary winding is con-nected across the supply while the secondary winding is connected in series with the line whose voltage is to be controlled. When the position of one winding is changed w.r.t. the other, the second- ary voltage injected into the line also changes. There are two types of induction regulators *viz.* singlephase and 3-phase.

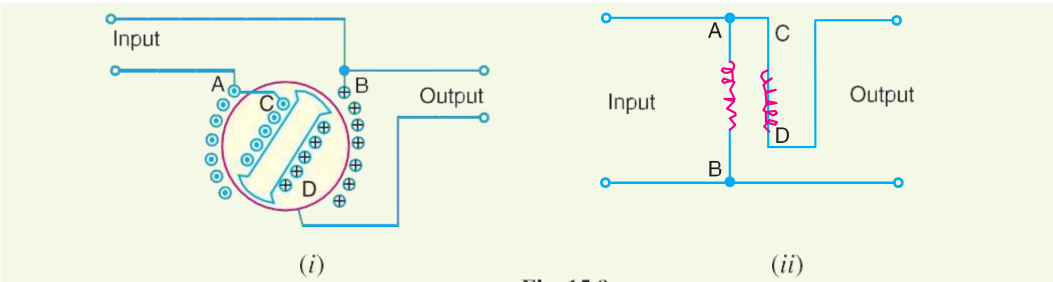


Fig. 15.9

(i) Single-phase induction regulator. A single phase induction regulator is illustrated in Fig. 15.9. In construction, it is similar to a single phase induction motor except that the rotor is not allowed to rotate continuously but can be adjusted in any position either manually or by a small motor. The primary winding *AB* is wound on the *stator and is connected across the supply line. The secondary winding *CD* is wound on the rotor and is connected in series with the line whose voltage isto be controlled.

The primary exciting current produces an alternating flux that induces an alternating voltage inthe secondary winding *CD*. The magnitude of voltage induced in the secondary depends upon its position *w.r.t.* the primary winding. By adjusting the rotor to a suitable position, the secondary volt- age can be varied from a maximum positive to a maximum negative value. In this way, the regulatorcan add or subtract from the circuit voltage according to the relative positions of the two windings. Owing to their greater flexibility, single phase regulators are frequently used for voltage control ofdistribution primary feeders.

(ii) Three-phase induction regulator. In construction, a 3-phase induction regulator is similar to a 3-phase induction motor with wound rotor except that the rotor is not allowed to rotate continuously but can be held in any position by means of a worm gear. The primary windings either in star or delta are wound on the stator and are connected across the supply. The secondary windings are wound on the rotor and the six terminals are brought out since these windings are to be connected in series with the line whose voltage is to be controlled.

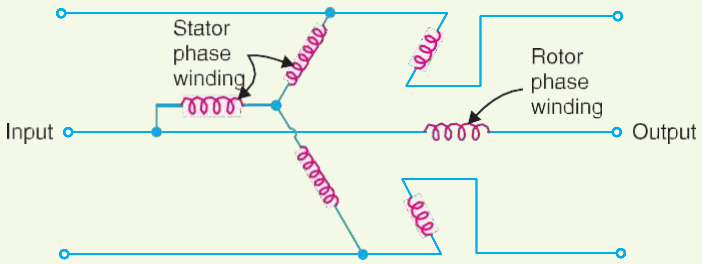


Fig. 15.10

* From electrical point of view, it is immaterial whether the rotor or stator carries the primary winding.

When polyphase currents flow through the primary wind- ings, a rotating field is set up which induces an e.m.f. in each phase of rotor winding. As the rotor is turned, the magnitude ofthe rotating flux is not changed; hence the rotor e.m.f. per phase remains constant. However, the variation of the position of therotor will affect the phase of the rotor e.m.f. *w.r.t.* the applied voltage as shown in Fig. 15.11. The input primary voltage perphase is V_p and the boost introduced by the regulator is V_r. The output voltage V is the vector sum of V_p and V_r. Three phase induction regulators are used to regulate the voltage of feeders and in connection with high voltage oil testing transformers.

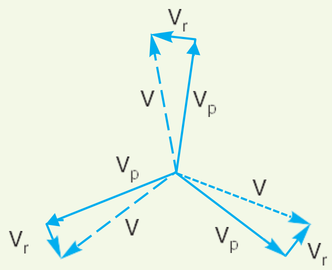


Fig. 15.11

15.12 Voltage Control by Synchronous Condenser

The voltage at the receiving end of a transmission line can be controlled by installing specially designed synchronous motors called *synchronous condensers at the receiving end of the line. Thesynchronous condenser supplies wattless leading kVA to the line depending upon the excitation of themotor. This wattless leading kVA partly or fully cancels the wattless lagging kVA of the line, thuscontrolling the voltage drop in the line. In this way, voltage at the receiving end of a transmission line can be kept constant as the load on the system changes.

Voltage Control

For simplicity, consider a short transmission line where the effects of capacitance are neglected. Therefore, the line has only resistance and inductance. Let V_1 and V_2 be the per phase sending endand receiving end voltages respectively. Let I_2 be the load current at a lagging power factor of cos ϕ_2.

(i) Without synchronous condenser. Fig. 15.12 (*i*) shows the transmission line with resistance *R* and inductive reactance *X* per phase. The load current I_2 can be resolved into two rectangular components *viz* I_p in phase with V_2 and I_q at right angles to V_2 [See Fig. 15.12 (*ii*)]. Each component will produce resistive and reactive drops ; the resistive drops being in phase with and the reactive drops in quadrature leading with the

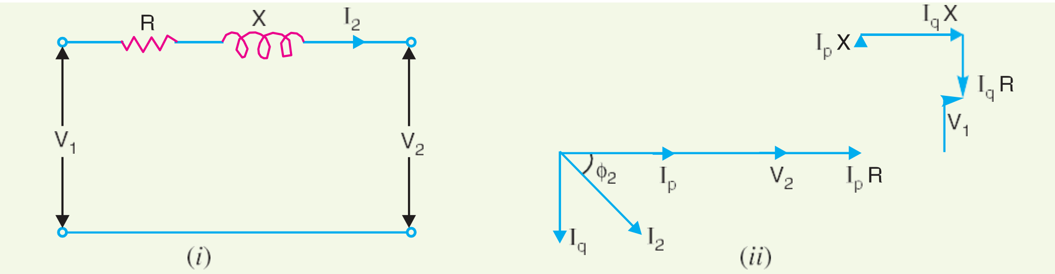


Fig. 15.12

corresponding currents. The vector addition of these voltage drops to V_2 gives the sending end voltage V_1.

(ii) With synchronous condenser. Now suppose that a synchronous condenser taking a leading current $**I_m$ is connected at the receiving end of the line. The vector diagram of the circuit becomes as shown in Fig. 15.13. Note that since I_m and I_q are in direct opposition and that I_m must be greater than I_q, the four drops due to these two currents simplify to :

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* By changing the excitation of a synchronous motor, it can be made to take a leading power factor. A synchronous motor at no load and taking a leading power factor is known as a *synchronous condenser*. It is so called because the characteristics of the motor then resemble with that of a condenser.

** Neglecting the losses of the synchronous condenser, I_m will lead V_2 by 90°.

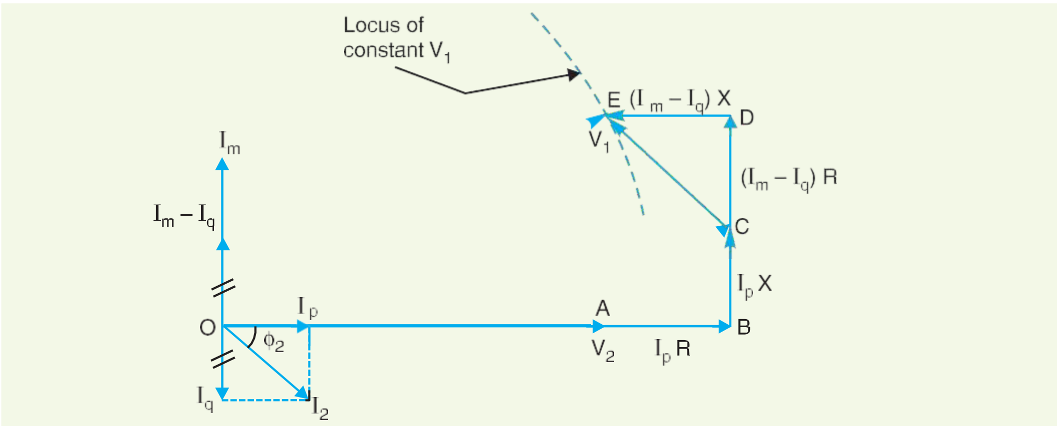

Fig. 15.13

$(I_m - I_q) R$ in phase with I_m

and $(I_m - I_q)\,X$ in quadrature leading with I_m

From the vector diagram, the relation between V_1 and V_2 is given by ;

$$OE^2 = (OA + AB - DE)^2 + (BC + CD)^2$$

or

$$V_1^2 = [V_2 + I_p R - (I_m - I_q) X]^2 + [I_p X + (I_m - I_q) R]^2$$

From this equation, the value of I_m can be calculated to obtain any desired ratio of V_1/V_2 for agiven load current and power factor.

$$\text{kVAR capacity of condenser} = \frac{3\, V_2\, I_m}{1000}$$

Synchronous Condenser

Example 15.1. *A load of 10,000 kW at a power factor of 0·8 lagging is supplied by a 3-phase line whose voltage has to be maintained at 33kV at each end. If the line resistance and reactance per phase are 5 Ω and 10 Ω respectively, calculate the capacity of the synchronous condenser to be installed for the purpose. Comment on the result.*

$$I_2 = \frac{10,000 \cdot 10^3}{\sqrt{3} \cdot 33 \cdot 10^3 \cdot 0{\cdot}8} = 218 \text{ A}$$

∴

$$I_p = I_2 \cos \phi_2 = 218 \cdot 0{\cdot}8 = 174{\cdot}4 \text{ A}$$

$$I_q = I_2 \sin \phi_2 = 218 \cdot 0{\cdot}6 = 130{\cdot}8 \text{ AR}$$

$= 5\,\Omega$; $X = 10\,\Omega$

Sending-end voltage/phase, V_1 = Receiving end voltage/phase (V_2)

$$= \frac{33 \cdot 10^3}{\sqrt{3}} = 19,053 \text{ V}$$

Let I_m be the current taken by the synchronous condenser. Referring to Fig. 15.13,

$$(19,053)^2 = [19,053 + 174{\cdot}4 \cdot 5 - 10\,(I_m - 130{\cdot}8)]^2 + [174{\cdot}4 \cdot 10 + (I_m - 130{\cdot}8)5]^2$$

Solving this equation, we get, $I_m = 231$ A

$$\text{Capacity of synchronous condenser} = \frac{3 V_2\, I_m}{1000} \text{ kVAR} = \frac{3 \cdot 19,053 \cdot 231}{1000} \text{ kVAR}$$

$= \mathbf{13,203\ kVAR}$

Comments. This example shows that kVA capacity of the synchronous condenser is consider- ably greater than the kVA capacity of the load *viz* 13203 against 10000/0·8 = 12,500. Since the costof synchronous condenser is usually very high, it would not be an economical proposition to have the same sending end and receiving end voltages. In practice, the synchronous condenser is operated insuch a way so as to allow a small drop in the line.

Example 15.2. *A 3-phase overhead line has resistance and reactance per phase of 5 Ω and 20 Ω respectively. The load at the receiving end is 25 MW at 33 kV and a power factor of 0·8 lagging. Find the capacity of the synchronous condenser required for this load condition if it is connected at the receiving end and the line voltages at both ends are maintained at 33 kV.*

$= 546 \cdot 8$ A

∴ $I_p = I_2 \cos \phi_2 = 546·8 \cdot 0·8 = 437·4$ A $I_q = I_2 \sin \phi_2 = 546·8 \cdot 0·6 = 328·1$ A $R = 5\,\Omega$; $X = 20\,\Omega$

Sending end voltage/phase, V_1 = Receiving end voltage/phase, V_2

$$= \frac{33 \cdot 10^3}{\sqrt{3}} = 19053 \text{ V}$$

Let I_m be the current taken by the synchronous condenser. Then,

$$V_1^2 = [V_2 + I_p R - (I_m - I_q) \cdot X]^2 + [I_p X + (I_m - I_q) R]^2$$

or $(19053)^2 = [19053 + 437·4 \cdot 5_m - (I - 328·1) \cdot 20]^2$

On solving this equation, we get, $I_m = 579·5$ A+ $[437·4 \cdot 20 + (I_m - 328·1) \cdot 5]^2$

Capacity of synchronous condenser $= \frac{3 V_2 I_m}{10^6}$ MVAR $= \frac{3 \cdot 19,053 \cdot 579·5}{10^6}$ = **33·13 MVAR**

TUTORIAL PROBLEMS

1. A 3-phase line having an impedance of (5 + *j* 20) ohms per phase delivers a load of 30 MW at a p.f. of 0·8 lagging and voltage 33 kV. Determine the capacity of the synchronous condenser required to be installed at the receiving end if voltage at the sending end is to be maintained at 33 kV.

[42·78 MVAR]

2. A 12500 kVA load is supplied at a power factor of 0·8 lagging by a 3-phase transmission line whose voltage is to be maintained at 33 kV at both ends. Determine the capacity of the synchronous condenser to be installed at the receiving end. The impedance of the line is (4 + *j* 12) ohms per phase.

[11490 kVAR]

Power Factor Improvement

Introduction

The electrical energy is almost exclusivelygenerated, transmitted and distributed in the form of alternating current. Therefore,

the question of power factor immediately comesinto picture. Most of the loads (*e.g.* induction motors, arc lamps) are inductive in nature and hence have low lagging power factor. The low power factor is highly undesirable as it causes anincrease in current, resulting in additional losses of active power in all the elements of power sys- tem from power station generator down to the utilisation devices. In order to ensure mostfavourable conditions for a supply system from engineering and economical standpoint, it is im- portant to have power factor as close to unity as possible. In this chapter, we shall discuss the various methods of power factor improvement.

6.1 Power Factor

The cosine of angle between voltage and currentin an a.c. circuit is known as **power factor.**

In an a.c. circuit, there is generally a phasedifference ϕ between voltage and current. The term $\cos\phi$ is called the power factor of the cir- cuit. If the circuit is inductive, the current lags behind the voltage and the power factor is referred to as lagging. However, in a capacitive circuit, current leads the volt-age and power factor is said to be leading.

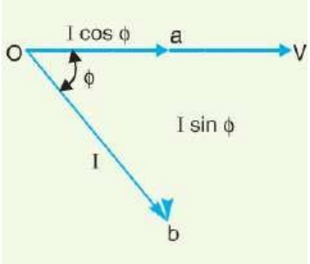

Fig. 6.1

Consider an inductive circuit taking a lagging current I from sup- ply voltage V; the angle of lag being ϕ. The phasor diagram of the circuit is shown in Fig. 6.1. The circuit current I can be resolved into two perpendicular components, namely ;

(a) $I\cos\phi$ in phase with V

(b) $I\sin\phi$ 90° out of phase with V

The component $I\cos\phi$ is known as active or wattful component, whereas component $I\sin\phi$ is called the reactive or wattless component. The reactive component is ameasure of the power factor. If the reactive component is small, the phase angle ϕ is small and hencepower factor $\cos\phi$ will be high. Therefore, a circuit having small reactive current (*i.e.*, $I\sin\phi$) willhave high power factor and *vice-versa*. It may be noted that value of power factor can never be morethan unity.

(i) It is a usual practice to attach the word 'lagging' or 'leading' with the numerical value of power factor to signify whether the current lags or leads the voltage. Thus if the circuit has a p.f. of 0·5 and the current lags the voltage, we generally write p.f. as 0·5 lagging.

(ii) Sometimes power factor is expressed as a percentage. Thus 0·8 lagging power factor may be expressed as 80% lagging.

6.2 Power Triangle

The analysis of power factor can also be made in terms of power drawn by the a.c. circuit. If each sideof the current triangle *oab* of Fig. 6.1 is multiplied by voltage V, then we get the power triangle *OAB*shown in Fig. 6.2 wher

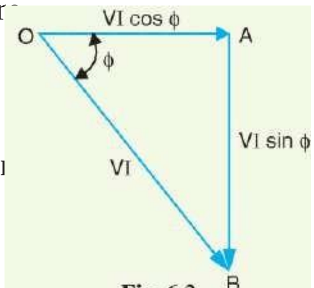

Fig. 6.2

$OA = VI\cos\phi$ and represents the *active power* in watts or kW

$AB = VI\sin\phi$ and represents the *reactive power* in VAR or kVAR

$OB = VI$ and represents the *apparent power* in VA or kVA

The following points may be noted form the power triai

(i) The apparent power in an a.c. circuit has two components *viz.*, active and reactive power at right angles to each other.

$$OB^2 = OA^2 + AB^2$$

or (apparent power)^2 = (active power)^2 + (reactive power)^{2}or

$$(\text{kVA})^2 = (\text{kW})^2 + (\text{kVAR})^2$$

(ii) Power factor, $\cos\phi = \dfrac{OA}{OB} = \dfrac{\text{active power}}{\text{apparent power}} = \dfrac{\text{kW}}{\text{kVA}}$

Thus the power factor of a circuit may also be defined as the ratio of active power to the apparent power. This is a perfectly general definition and can be applied to all cases, what-ever be the waveform.

(iii) The lagging* reactive power is responsible for the low power factor. It is clear from the power triangle that smaller the reactive power component, the higher is the power factor of the circuit.

$$\text{kVAR} = \text{kVA} \sin\phi = \frac{\text{kW}}{\cos\phi} \sin\phi$$

$$\therefore \quad \text{kVAR} = \text{kW} \tan\phi$$

* If the current lags behind the voltage, the reactive power drawn is known as lagging reactive power. However, if the circuit current leads the voltage, the reactive power is known as leading reactive power.

(iv) For leading currents, the power triangle becomes reversed. This fact provides a key to the power factor improvement. If a device taking leading reactive power (*e.g.* capacitor) is connected in parallel with the load, then the lagging reactive power of the load will be partly neutralised, thus improving the power factor of the load.

(v) The power factor of a circuit can be defined in one of the following three ways :

(a) Power factor = $\cos\phi$ = cosine of angle between V and I

(b) Power factor $= \dfrac{R}{Z} = \dfrac{\text{Resistance}}{\text{Impedance}}$

(c) Power factor $= \dfrac{VI\cos\phi}{VI} = \dfrac{\text{Active power}}{\text{Apparent Power}}$

(vi) The reactive power is neither consumed in the circuit nor it does any useful work. It merely flows back and forth in both directions in the circuit. A wattmeter does not measure reactive power.

Illustration. Let us illustrate the power relations in an a.c. circuit with an example. Suppose acircuit draws a current of 10 A at a voltage of 200 V and its p.f. is 0·8 lagging. Then,

Apparent power $= VI = 200 \cdot 10 = 2000$ VA

Active power $= VI\cos\phi = 200 \cdot 10 \cdot 0{\cdot}8 = 1600$

W Reactive power $= VI\sin\phi = 200 \cdot 10 \cdot 0{\cdot}6 =$ 1200 VAR

The circuit receives an apparent power of 2000 VA and is able to convert only 1600 watts intoactive power. The reactive power is 1200 VAR and does no useful work. It merely flows into and out of the circuit periodically. In fact, reactive power is a liability on the source because the source has tosupply the additional current (*i.e.*, $I\sin\phi$).

6.3 Disadvantages of Low Power Factor

The power factor plays an importance role in a.c. circuits since power consumed depends upon thisfactor.

$$P = V_L I_L \cos\phi \qquad \text{(For single phase supply)}$$

$$\therefore \quad I_L = \frac{P}{V_L \cos\phi} \qquad ...(i)$$

$$P = \sqrt{3}\, V_L I_L \cos\phi \qquad \text{(For 3 phase supply)}$$

$$\therefore \quad I_L = \frac{P}{\sqrt{3}\, V_L \cos\phi}$$

It is clear from above that for fixed power and voltage, the load current is inversely proportionalto the power factor. Lower the power factor, higher is the load current and *vice-versa*. A power factor less than unity results in the following

disadvantages :

(i) Large kVA rating of equipment. The electrical machinery (*e.g.*, alternators, transformers, switchgear) is always rated in *kVA.

Now, $$\text{kVA} = \frac{\text{kW}}{\cos\phi}$$

It is clear that kVA rating of the equipment is inversely proportional to power factor. The smaller the power factor, the larger is the kVA rating. Therefore, at low power factor, the kVA rating of theequipment has to be made more, making the equipment larger and expensive.

(ii) Greater conductor size. To transmit or distribute a fixed amount of power at constant voltage, the conductor will have to carry more current at low power factor. This necessitates

* The electrical machinery is rated in kVA because the power factor of the load is not known when the machinery is manufactured in the factory.

large conductor size. For example, take the case of a single phase a.c. motor having an input of 10 kW on full load, the terminal voltage being 250 V. At unity p.f., the input full load current would be 10,000/250 = 40 A. At 0·8 p.f; the kVA input would be 10/0·8 = 12·5 andthe current input 12,500/250 = 50 A. If the motor is worked at a low power factor of 0·8, thecross-sectional area of the supply cables and motor conductors would have to be based upon a current of 50 A instead of 40 A which would be required at unity power factor.

(iii) Large copper losses. The large current at low power factor causes more I^2R losses in all the elements of the supply system. This results in poor efficiency.

(iv) Poor voltage regulation. The large current at low lagging power factor causes greater voltage drops in alternators, transformers, transmission lines and distributors. This results in the decreased voltage available at the supply end, thus impairing the performance of utilisation devices. In order to keep the receiving end voltage within permissible limits, extra equipment (*i.e.*, voltage regulators) is required.

(v) Reduced handling capacity of system. The lagging power factor reduces the handling capacity of all the elements of the system. It is because the reactive component of current prevents the full utilisation of installed capacity.

The above discussion leads to the conclusion that low power factor is an objectionable feature inthe supply system

6.4 Causes of Low Power Factor

Low power factor is undesirable from economic point of view. Normally, the power factor of the whole load on the supply system in lower than 0·8. The following are the causes of low power factor:

(i) Most of the a.c. motors are of induction type (1ϕ and 3ϕ induction motors) which have low lagging power factor. These motors work at a power factor which is extremely small on light load (0·2 to 0·3) and rises to 0·8 or 0·9 at full load.

(ii) Arc lamps, electric discharge lamps and industrial heating furnaces operate at low lagging power factor.

(iii) The load on the power system is varying ; being high during morning and evening and low at other times. During low load period, supply voltage is increased which increases the magnetisation current. This results in the decreased power factor.

6.5 Power Factor Improvement

The low power factor is mainly due to the fact that most of the power loads are inductive and, there- fore, take lagging currents. In order to improve the power factor, some device taking leading powershould be connected in parallel with the load. One of such devices can be a capacitor. The capacitordraws a leading current

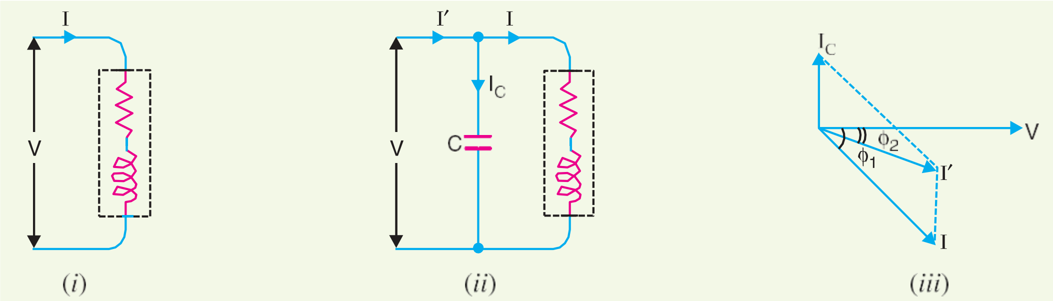

Fig. 6.3

and partly or completely neutralises the lagging reactive component of loadcurrent. This raises the power factor of the load.

Illustration. To illustrate the power factor improvement by a capacitor, consider a single *phase load taking lagging current I at a power factor $\cos\phi_1$ as shown in Fig. 6.3.

The capacitor C is connected in parallel with the load. The capacitor draws current I_C which leads the supply voltage by 90°. The resulting line current I' is the phasor sum of I and I_C and its angle of lag is ϕ_2 as shown in the phasor diagram of Fig. 6.3. (*iii*). It is clear that ϕ_2 is less than ϕ_1, so thatcos ϕ_2 is greater than $\cos\phi_1$. Hence, the power factor of the load is improved. The following pointsare worth noting :

(i) The circuit current I' after p.f. correction is less than the original circuit current I.

(ii) The active or wattful component remains the same before and after p.f. correction because only the lagging reactive component is reduced by the capacitor.

$$\therefore \quad I\cos\phi_1 = I'\cos\phi_2$$

(iii) The lagging reactive component is reduced after p.f. improvement and is equal to the difference between lagging reactive component of load ($I\sin\phi_1$) and capacitor current (I_C) *i.e.*,

$$I'\sin\phi_2 = I\sin\phi_1 - I_C$$

(iv) As

$$I\cos\phi_1 = I'\cos\phi_2$$

$$\therefore \quad VI\cos\phi_1 = VI'\cos\phi_2 \qquad \text{[Multiplying by } V\text{]}$$

Therefore, active power (kW) remains unchanged due to power factor improvement.

(v)

$$I'\sin\phi_2 = I\sin\phi_1 - I_C$$

$$\therefore \quad VI'\sin\phi_2 = VI\sin\phi_1 - VI_C \qquad \text{[Multiplying by } V\text{]}$$

i.e., Net kVAR *after* p.f. correction = Lagging kVAR *before* p.f. correction – leading kVAR of equipment

6.6 Power Factor Improvement Equipment

Normally, the power factor of the whole load on a large generating station is in the region of 0·8 to0·9. However, sometimes it is lower and in such cases it is generally desirable to take special steps toimprove the power factor. This can be achieved by the following equipment :

1. Static capacitors. **2.** Synchronous condenser. **3.** Phase advancers.

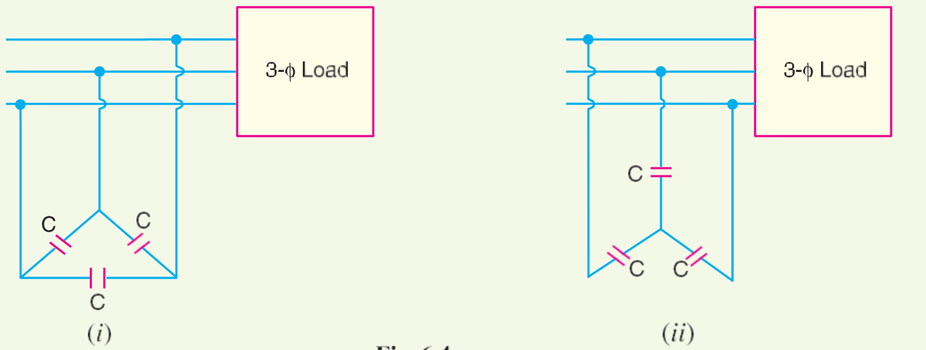

Fig. 6.4

1. Static capacitor. The power factor can be improved by connecting capacitors in parallel with the equipment operating at lagging power factor. The capacitor (generally known as static**

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* The treatment can be used for 3-phase balanced loads *e.g.*, 3-ϕ induction motor. In a balanced 3-ϕ load, analysis of one phase leads to the desired results.

** To distinguish from the so called *synchronous condenser* which is a synchrnous motor running at no load and taking leading current.

capacitor) draws a leading current and partly or completely neutralises the lagging reactive compo-nent of load current. This raises the power factor of the load. For three-phase loads, the capacitorscan be connected in delta or star as shown in Fig. 6.4. Static capacitors are invariably used for powerfactor improvement in factories.

Advantages

(i) They have low losses.

(ii) They require little maintenance as there are no rotating parts.

(iii) They can be easily installed as they are light and require no foundation.

(iv) They can work under ordinary atmospheric conditions.

Disadvantages

(i) They have short service life ranging from 8 to 10 years.

(ii) They are easily damaged if the voltage exceeds the rated value.

(iii) Once the capacitors are damaged, their repair is uneconomical.

2. **Synchronous condenser.** A synchronous motor takes a leading current when over-excited and, therefore, behaves as a capacitor. An over-excited synchronous motor running on no load is known as *synchronous condenser*. When such a machine is connected in parallel with the supply, it takes a leading current which partly neutralises the lagging reactive component of the load. Thus the power factor is improved.

Fig 6.5 shows the power factor improvement by synchronous condenser method. The 3ϕ load takescurrent I_L at low lagging power factor cos ϕ_L. The synchronous condenser takes a current I_m which leads the voltage by an angle ϕ_m*. The resultant current I is the phasor sum of I_m and I_L and lags behind the voltage by an angle ϕ. It is clear that ϕ is less than ϕ_L so that cos ϕ is greater than cos ϕ_L.Thus the power factor is increased from cos ϕ_L to cos ϕ. Synchronous condensers are generally used at

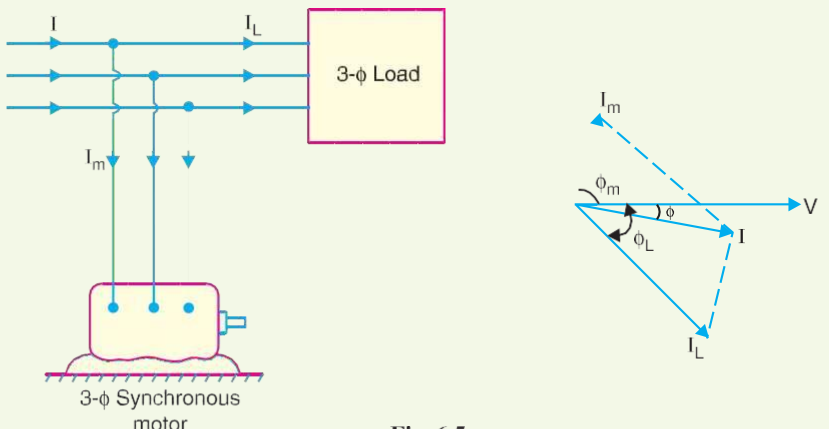


Fig. 6.5

major bulk supply substations for power factor improvement.

Advantages

(i) By varying the field excitation, the magnitude of current drawn by the motor can be changed by any amount. This helps in achieving stepless † control of power factor.

* If the motor is ideal *i.e.*, there are no losses, then $\phi_m = 90°$. However, in actual practice, losses do occur in

the motor even at no load. Therefore, the currents I_m leads the voltage by an angle less than 90 .

† The *p.f.* improvement with capacitors can only be done in steps by switching on the capacitors in various groupings. However, with synchronous motor, any amount of capacitive reactance can be provided by changing the field excitation.

(ii) he motor windings have high thermal stability to short circuit currents.

(iii) The faults can be removed easily.

Disadvantages

(i) There are considerable losses in the motor.

(ii) The maintenance cost is high.

(iii) It produces noise.

(iv) Except in sizes above 500 kVA, the cost is greater than that of static capacitors of the same rating.

(v) As a synchronous motor has no self-starting torque, therefore, an auxiliary equipment has to be provided for this purpose.

Note. The reactive power taken by a synchronous motor depends upon two factors, the d.c. field excitation and the mechanical load delivered by the motor. Maximum leading power is taken by a synchronous motor with maximum excitation and zero load.

Synchronous Condenser

Static Capacitor

3. Phase advancers. Phase advancers are used to improve the power factor of induction motors. The low power factor of an induction motor is due to the fact that its stator winding draws exciting current which lags behind the supply voltage by 90°. If the exciting ampere turns can be provided from some other a.c. source, then the stator winding will be relieved of exciting current and the power factor of the motor can be improved. This job is accomplished by the phase advancer which is simply an a.c. exciter. The phase advancer is mounted on the same shaft as the main motor and is connected in the rotor circuit of the motor. It provides exciting ampere turns to the rotor circuit at slip frequency. By providing more ampere turns than required, the induction motor can be made to operate on leading power factor like an over-excited synchronous motor.

Phase advancers have two principal advantages. Firstly, as the exciting ampere turns are sup- plied at slip frequency, therefore, lagging kVAR drawn by the motor are considerably reduced. Sec-ondly, phase advancer can be conveniently used where the use of synchronous motors is unadmissible. However, the major disadvantage of phase advancers is that they are not economical for motors below 200 H.P.

6.7 Calculations of Power Factor Correction

Consider an inductive load taking a lagging current I at a power factor cos ϕ_1. In order to improve the power factor of this circuit, the remedy is to connect such an equipment in parallel with the load which takes a leading reactive component and partly cancels the lagging reactive component of the load. Fig. 6.6 (*i*) shows a capacitor connected across the load. The capacitor takes a current I_C which leads the supply voltage V by 90°. The current I_C partly cancels the lagging reactive component of the load current as shown in the phasor diagram in Fig. 6.6 (*ii*). The resultant circuit current becomes I' and its angle of lag is ϕ_2. It is clear that ϕ_2 is less than ϕ_1 so that

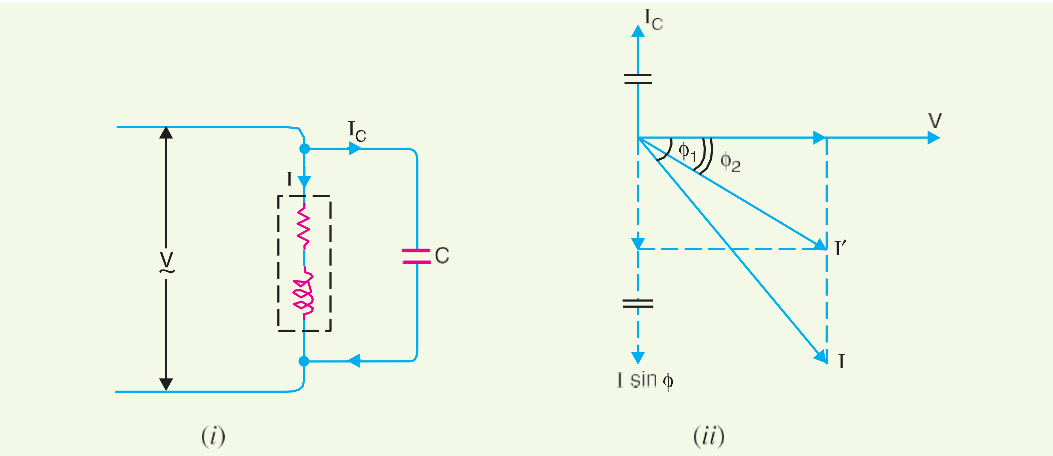


Fig. 6.6

new p.f. cos ϕ_2 is more than the previous p.f. cos ϕ_1.

From the phasor diagram, it is clear that after p.f. correction, the lagging reactive component of the load is reduced to $I'\sin\phi_2$.

Obviously, $I'\sin\phi_2 = I\sin\phi_1 - I_C$

or $I_C = I\sin\phi_1 - I'\sin\phi_2$

∴ Capacitance of capacitor to improve p.f. from cos ϕ_1 to cos ϕ_2

$$= \frac{I_C}{\omega V}$$

G $= _{\omega C}$KJ

Power Factor Improvement

H $\because X_C = _I$

Power triangle. The power factor correction can also be illustrated from power triangle. Thusreferring to Fig. 6.7, the power triangle *OAB* is for the power factor cos ϕ_1, whereas power triangle*OAC* is for the improved power factor cos ϕ_2. It may be seen that active power (*OA*) does not change with power factor improve-ment. However, the lagging kVAR of the load is reduced by the p.f. correction equipment, thus improving the p.f. to cos ϕ_2.

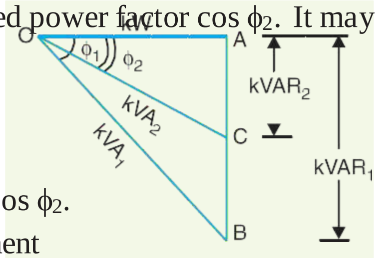


Fig. 6.7

Leading kVAR supplied by p.f. correction equipment

$$= BC = AB - AC$$
$$= \text{kVAR}_1 - \text{kVAR}_2$$
$$= OA\,(\tan\phi_1 - \tan\phi_2)$$
$$= \text{kW}\,(\tan\phi_1 - \tan\phi_2)$$

Knowing the leading kVAR supplied by the p.f. correction equipment, the

Example 6.1 *An alternator is supplying a load of 300 kW at a p.f. of 0·6 lagging. If the power factor is raised to unity, how many more kilowatts can alternator supply for the same kVA loading ?*

desired results can beobtained.

Solution :

$$\text{kVA} = \frac{\text{kW}}{\cos\phi} = \frac{300}{0 \cdot 6} = 500 \text{ kVA}$$

kW at 0·6 p.f. = 300 kW

kW at 1 p.f. = 500 · 1 = 500 kW

∴ Increased power supplied by the alternator

= 500 – 300 = **200 kW**

Note the importance of power factor improvement. When the p.f. of the alternator is unity, the500 kVA are also 500 kW and the engine driving the alternator has to be capable of developing thispower together with the losses in the alternator. But when the power factor of the load is 0·6, the power is only 300 kW. Therefore,

Example 6.2 *A single phase motor connected to 400 V, 50 Hz supply takes 31·7A at a power factor of 0·7 lagging. Calculate the capacitance required in parallel with the motor to raise the power factor to 0·9 lagging.*

the engine is developing only 300 kW, though the alternator is supplying its rated output of 500 kVA.

Solution : The circuit and phasor diagrams are shown in Figs. 6.8 and 6.9 respectively. Heremotor *M* is taking a current I_M of 31·7A. The current I_C taken by the capacitor must be such that when combined with I_M, the resultant current *I* lags the voltage by an angle φ where cos φ = 0·9.

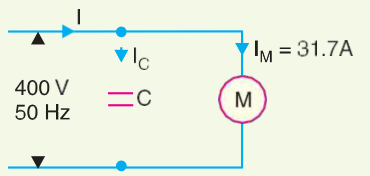


Fig. 6.8

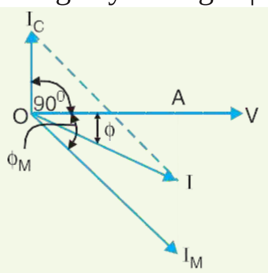


Fig. 6.9

Referring to the phasor diagram in Fig. 6.9,

Active component of $I_M = I_M \cos\phi_M = 31 \cdot 7 \cdot 0 \cdot 7 =$ 22·19AActive component of $I = I \cos\phi = I \cdot 0 \cdot 9$

These components are represented by *OA* in Fig. 6.9.

$$\therefore \quad I = \frac{22 \cdot 19}{0 \cdot 9} = 24 \cdot 65 \text{A}$$

Reactive component of $I_M = I_M \sin\phi_M = 31 \cdot 7 \cdot 0 \cdot 714$* =

22·6AReactive component of $I = I \sin\phi = 24 \cdot 65 \sqrt{1 - (0 \cdot 9)^2}$

= 24·65 · 0·436 = 10·75 A

It is clear from Fig. 6.9 that :

I_C = Reactive component of I_M – Reactive component of *I*

= 22·6 – 10·75 = 11·85A

But $$I_C = \frac{V}{X_C} = V \cdot 2\pi f C$$

or $$11{\cdot}85 = 400 \cdot 2\pi \cdot 50 \cdot C$$

$\therefore$ $$C = 94{\cdot}3 \cdot 10^{-6}\text{ F} = \mathbf{94{\cdot}3\ \mu F}$$

---

* $\sin \phi_M = \sqrt{1 - \cos^2 \phi_M} = \sqrt{1-(0{\cdot}7)^2} = 0{\cdot}714$

Note the effect of connecting a 94·3 ∝F capacitor in parallel with the motor. The current taken from the supply is reduced from 31·7 A to 24·65 A without altering the current or power taken by themotor. This enables an economy to be affected in the

Example 6.3 *A single phase a.c. generator supplies the following loads :*

(i) *Lighting load of 20 kW at unity power factor.*

(ii) *Induction motor load of 100 kW at p.f. 0·707 lagging.*

(iii) *Synchronous motor load of 50 kW at p.f. 0·9 leading.*

Calculate the total kW and kVA delivered by the generator and the power factor at which it works.

size of generating plant and in the cross- sectional area of the conductors.

Solution : Using the suffixes 1, 2 and 3 to indicate the different loads, we have,

$$kVA_1 = \frac{kW_1}{\cos\phi_1} = \frac{20}{1} = 20 \text{ kVA}$$

$$kVA_2 = \frac{kW_2}{\cos\phi_2} = \frac{100}{0 \cdot 707} = 141{\cdot}4 \text{ kVA}$$

$$kVA_3 = \frac{kW_3}{\cos\phi_3} = \frac{50}{0 \cdot 9} = 55{\cdot}6 \text{ kVA}$$

These loads are represented in Fig. 6.10. The three kVAs' are not in phase. In order to find thetotal kVA, we resolve each kVA into rectangular components – kW

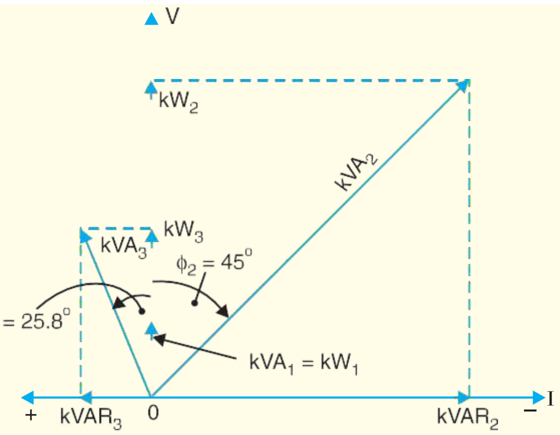


Fig. 6.10

and kVAR as shown in Fig. 6.10.The total kW and kVAR may then be combined to obtain total kVA.

$kVAR_1 = kVA_1 \sin\phi_1 = 20 \cdot 0 = 0$

$kVAR_2 = kVA_2 \sin\phi_2 = -141{\cdot}4 \cdot 0{\cdot}707 = -100$

$kVARkVAR_3 = kVA_3 \sin\phi_3 = +55{\cdot}6 \cdot 0{\cdot}436 = +24{\cdot}3$ kVAR

Note that $kVAR_2$ and $kVAR_3$ are in opposite directions ; $kVAR_2$ being a lagging while $kVAR_3$ being a leading kVAR.

$$\text{Total kW} = 20 + 100 + 50 = \mathbf{170\ kW}$$

$$\text{Total kVAR} = 0 - 100 + 24{\cdot}3 = -75{\cdot}7\ \text{kVAR}$$

$$\text{Total kVA} = \sqrt{(\text{kW})^2 + (\text{kVAR})^2} = \sqrt{(170)^2 + (75{\cdot}7)^2} = \mathbf{186\ kVA}$$

$$\text{Power factor} = \frac{\text{Total kW}}{\text{Total kVA}} = \frac{170}{186} = \mathbf{0{\cdot}914\ lagging}$$

The power factor must be lagging since the resultant kVAR is lagging.

Example 6.4 *A 3-phase, 5 kW induction motor has a p.f. of 0·75 lagging. A bank of capacitors is connected in delta across the supply terminals and p.f. raised to 0·9 lagging. Determine the kVAR rating of the capacitors connected in each phase.*

Solution :

Original p.f., $\cos\phi_1 = 0{\cdot}75$ lag ; Motor input, $P = 5$ kW

Final p.f., $\cos\phi_2 = 0{\cdot}9$ lag ; Efficiency, $\eta = 100\,\%$ (assumed)

$\phi_1 = \cos^{-1}(0{\cdot}75) = 41{\cdot}41^\circ$; $\tan\phi_1 = \tan 41{\cdot}41^\circ = 0{\cdot}8819$

$\phi_2 = \cos^{-1}(0{\cdot}9) = 25{\cdot}84^\circ$; $\tan\phi_2 = \tan 25{\cdot}84^\circ = 0{\cdot}4843$

Leading kVAR taken by the condenser bank

$= P(\tan\phi_1 - \tan\phi_2)$

$= 5\,(0{\cdot}8819 - 0{\cdot}4843) = 1{\cdot}99$ kVAR

∴ Rating of capacitors connected in each phase

$= 1{\cdot}99/3 =$ **0·663 kVAR**

Example 6.5 *A 3-phase, 50 Hz, 400 V motor develops 100 H.P. (74·6 kW), the power factor being 0·75 lagging and efficiency 93%. A bank of capacitors is connected in delta across the supply terminals and power factor raised to 0·95 lagging. Each of the capacitance units is built of 4 similar 100 V capacitors. Determine the capacitance of each capacitor.*

Solution :

Original p.f., $\cos\phi_1 = 0{\cdot}75$ lag ; Final p.f., $\cos\phi_2 = 0{\cdot}95$ lagMotor input, P = output/η = 74·6/0·93 = 80 kW

$\phi_1 = \cos^{-1}(0{\cdot}75) = 41{\cdot}41^\circ$

$\tan\phi_1 = \tan 41{\cdot}41^\circ = 0{\cdot}8819$

$\phi_2 = \cos^{-1}(0{\cdot}95) = 18{\cdot}19^\circ$

$\tan\phi_2 = \tan 18{\cdot}19^\circ = 0{\cdot}3288$

Leading kVAR taken by the condenser bank

$= P(\tan\phi_1 - \tan\phi_2)$

$= 80\,(0{\cdot}8819 - 0{\cdot}3288) = 44{\cdot}25$ kVAR

Leading kVAR taken by each of three sets

$= 44{\cdot}25/3 = 14{\cdot}75$ kVAR ... (*i*)

Fig. 6.11 shows the delta* connected condenser bank. Let C farad be the capacitance of 4capacitors in each phase.

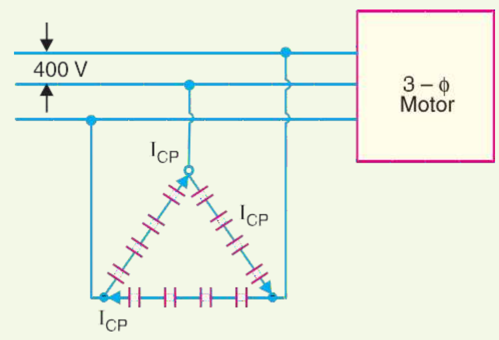


Fig. 6.11

Phase current of capacitor is

$I_{CP} = V_{ph}/X_C = 2\pi f C V_{ph}$

$= 2\pi \cdot 50 \cdot C \cdot 400$

$= 1{,}25{,}600\, C$ amperes

kVAR/phase $= \dfrac{V_{ph}\, I_{CP}}{1000}$

$= \dfrac{400 \cdot 1{,}25{,}600\, C}{1000}$

$= 50240\, C$ ... (*ii*)

* In practice, capacitors are always connected in delta since the capacitance of the capacitor required is one-third of that required for star connection.

Equating exps. (*i*) and (*ii*), we get,

$$50240\ C = 14{\cdot}75$$

$$\therefore \quad C = 14{\cdot}75/50{,}240 = 293{\cdot}4 \cdot 10^{-6}\ \text{F} = 293{\cdot}4 \propto \text{F}$$

Since it is the combined capacitance of four equal capacitors joined in series,

∴ Capacitance of each capacitor = 4 · 293·4 = **1173·6 ∝F**

Example 6.6. *The load on an installation is 800 kW, 0·8 lagging p.f. which works for 3000 hours per annum. The tariff is Rs 100 per kVA plus 20 paise per kWh. If the power factor is improved to 0·9 lagging by means of loss-free capacitors costing Rs 60 per kVAR, calculate the annual saving effected. Allow 10% per annum for interest and depreciation on capacitors.*

Solution.

Load, P = 800 kW

$\cos\phi_1 = 0{\cdot}8$; $\tan\phi_1 = \tan(\cos^{-1} 0{\cdot}8) = 0{\cdot}75$

$\cos\phi_2 = 0{\cdot}9$; $\tan\phi_2 = \tan(\cos^{-1} 0{\cdot}9) = 0{\cdot}4843$

Leading kVAR taken by the capacitors

$$= P(\tan\phi_1 - \tan\phi_2) = 800\,(0{\cdot}75 - 0{\cdot}4843) = 212{\cdot}56$$

Annual cost before p.f. correction

Max. kVA demand = 800/0·8 = 1000

kVA demand charges = Rs 100 · 1000 = Rs 1,00,000

Units consumed/year = 800 · 3000 = 24,00,000 kWh

Energy charges/year = Rs 0·2 · 24,00,000 = Rs 4,80,000

Total annual cost = Rs (1,00,000 + 4,80,000) = Rs 5,80,000

Annual cost after p.f. correction

Max. kVA demand = 800/0·9 = 888·89

kVA demand charges = Rs 100 · 888·89 = Rs 88,889

Energy charges = Same as before *i.e.*, Rs 4,80,000

Capital cost of capacitors = Rs 60 · 212·56 = Rs 12,750

Annual interest and depreciation = Rs 0·1 · 12750 = Rs 1275

Total annual cost = Rs (88,889 + 4,80,000 + 1275) = Rs 5,70,164

∴ Annual saving = Rs (5,80,000 – 5,70,164) = **Rs 9836**

Example 6.7. *A factory takes a load of 200 kW at 0·85 p.f. lagging for 2500 hours per annum. The traiff is Rs 150 per kVA plus 5 paise per kWh consumed. If the p.f. is improved to 0·9 lagging by means of capacitors costing Rs 420 per kVAR and having a power loss of 100 W per kVA, calculate the annual saving effected by their use. Allow 10% per annum for interest and depreciation.*

Solution :

Factory load, P_1 = 200 kW

$\cos\phi_1 = 0{\cdot}85$; $\tan\phi_1 = 0{\cdot}62$

$\cos\phi_2 = 0{\cdot}9$; $\tan\phi_2 = 0{\cdot}4843$

Suppose the leading kVAR taken by the capacitors is x.

∴ Capacitor loss $= \dfrac{100 \cdot x}{1000} = 0{\cdot}1\,x$ kW

Total power, $P_2 = (200 + 0{\cdot}1x)$ kW

Leading kVAR taken by the capacitors is

$$x = P_1 \tan\phi_1 - P_2 \tan\phi_2$$

$$= 200 \cdot 0{\cdot}62 - (200 + 0{\cdot}1x) \cdot 0{\cdot}4843$$

or $x = 124 - 96{\cdot}86 - 0{\cdot}04843\,x$

$\therefore$ $x = 27{\cdot}14/1{\cdot}04843 = 25{\cdot}89$ kVAR

Annual cost before p.f. improvement

Max. kVA demand = 200/0.85 = 235.3

kVA demand charges = Rs 150 · 235·3 = Rs 35,295Units consumed/year = 200 · 2500 = 5,00,000 kWh

Energy charges = Rs 0·05 · 5,00,000 = Rs 25,000 Total annual cost = Rs (35,295 + 25,000) = Rs 60,295

Annual cost after p.f. improvement

Max. kVA demand = 200/0·9 = 222·2

kVA demand charges = Rs 150 · 222·2 = Rs 33,330 Energy charges = same as before *i.e.*, Rs 25,000

Annual interest and depreciation = Rs 420 · 25·89 · 0·1 = Rs 1087

Annual energy loss in capacitors = 0·1 x · 2500 = 0·1 · 25·89 · 2500 = 6472 kWhAnnual cost of losses occurring in capacitors

= Rs 0·05 · 6472 = Rs 323

$\therefore$ Total annual cost = Rs (33,330 + 25,000 + 1087 + 323) = Rs 59,740Annual saving = Rs (60,295 – 59,740) = **Rs 555**

Example 6.8. *A factory operates at 0·8 p.f. lagging and has a monthly demand of 750 kVA. The monthly power rate is Rs 8·50 per kVA. To improve the power factor, 250 kVA capacitors are installed in which there is negligible power loss. The installed cost of equipment is Rs 20,000 and fixed charges are estimated at 10% per year. Calculate the annual saving effected by the use of capacitors.*

Solution.

Monthly demand is 750 kVA.

$\cos\phi = 0{\cdot}8$; $\sin\phi = \sin(\cos^{-1} 0{\cdot}8)$

= 0·6kW component of demand = kVA · cos ϕ = 750 · 0·8 = 600

kVAR component of demand = kVA · sin ϕ = 750 · 0·6 = 450

Leading kVAR taken by the capacitors is 250 kVAR. Therefore, net kVAR after p.f. improve-ment is 450 – 250 = 200.

$\therefore$ kVA after p.f. improvement = $\sqrt{(600)^2 + (200)^2}$ = 632·45

Reduction in kVA = 750 – 632·45 =

117·5 Monthly saving on kVA charges = Rs 8·5 · 117·5 = Rs 998·75

Yearly saving on kVA charges = Rs 998·75 · 12 = Rs 11,985Fixed charges/year = Rs 0·1 · 20,000

= Rs 2000

Net annual saving = Rs (11,985 – 2000) = **Rs 9,985**

Example 6.9. *A synchronous motor improves the power factor of a load of 200 kW from 0.8 lagging to 0.9 lagging. Simultaneously the motor carries a load of 80 kW. Find* (*i*) *the leading kVAR taken by the motor* (*ii*) *kVA rating of the motor and* (*iii*) *power factor at which the motor operates.*

Solution.

Load, P_1 = 200 kW ; Motor load, P_2 = 80 kW

p.f. of load, $\cos\phi_1 = 0\cdot8$ lag

p.f. of combined load, $\cos\phi_2 = 0\cdot9$ lag

Combined load, $P = P_1 + P_2 = 200 + 80 = 280$ kWIn Fig. 6.12, Δ *OAB* is the power triangle for load,

Δ *ODC* for combined load and Δ *BEC* for the motor.

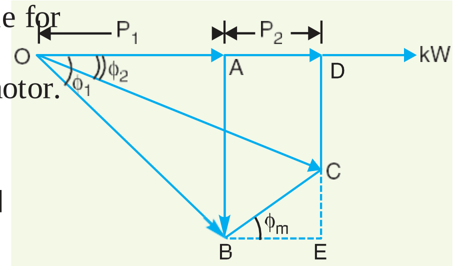


Fig. 6.12

(i) Leading kVAR taken by the motor

$= CE = DE - DC = AB - DC$

[$AB = DE$]

$= P_1 \tan \phi_1 - P^* \tan \phi_2$

$= 200 \tan(\cos^{-1} 0{\cdot}8) - 280 \tan(\cos^{-1} 0{\cdot}9)$

$= 200 \cdot 0{\cdot}75 - 280 \cdot 0{\cdot}4843$

= **14·4 kVAR**

(ii) kVA rating of the motor $= BC = \sqrt{(BE)^2 + (EC)^2} = \sqrt{(80)^2 + (14.4)^2}$ = **81·28 kVA**

(iii) p.f. of motor, $\cos \phi_m = \frac{\text{Motor kW}}{\text{Motor kVA}} = \frac{80}{81{\cdot}28}$ = **0·984 leading**

Example 6.10. *A factory load consists of the following :*

(i) an induction motor of 50 H.P. (37·3 kW) with 0·8 p.f. and efficiency 0·85.

(ii) a synchronous motor of 25 H.P. (18·65 kW) with 0·9 p.f. leading and efficiency 0·9.

(iii) lighting load of 10 kW at unity p.f.

Find the annual electrical charges if the tariff is Rs 60 per kVA of maximum demand per annum plus 5 paise per kWh ; assuming the load to be steady for 2000 hours in a year.

Solution.

Input power to induction motor = 37·3/0·85 = 43·88 kW

Lagging kVAR taken by induction motor = $43{\cdot}88 \tan(\cos^{-1} 0{\cdot}8) = 32{\cdot}91$

Input power to synchronous motor

= 18·65/0·9 = 20·72 kW

Leading kVAR taken by synchronous motor

$= 20{\cdot}72 \tan(\cos^{-1} 0{\cdot}9) =$ 10 Since lighting load works at unity p.f., its lagging kVAR = 0.

Net lagging kVAR = 32·91 – 10 = 22·91

Total active power = 43·88 + 20·72 + 10 = 74·6 kW

$$\text{Total kVA} = \sqrt{(74 \cdot 6)^2 + (22 \cdot 91)^2} = 78$$

Annual kVA demand charges = Rs 60 · 78 = Rs 4,680

Energy consumed/year = 74·6 · 2000 = 1,49,200

kWh Annual Energy charges = Rs 0·05 · 1,49,200 = Rs 7,460

Total annual bill = kVA demand charges + Energy charges

= Rs (4680 + 7460) = **Rs 12,140**

Example 6.11. *A supply system feeds the following loads* (*i*) *a lighting load of 500 kW* (*ii*) *a load of 400 kW at a p.f. of 0·707 lagging* (*iii*) *a load of 800 kW at a p.f. of 0·8 leading* (*iv*) *a load of 500 kW at a p.f. 0·6 lagging* (*v*) *a synchronous motor driving a 540 kW d.c. generator and having an overall efficiency of 90%. Calculate the power factor of synchronous motor so that the station power factor may become unity.*

* In right angled triangle *OAB*, $AB = P_1 \tan \phi_1$
In right angled triangle *ODC*, $DC = OD \tan \phi_2 = (P_1 + P_2) \tan \phi_2 = P \tan \phi_2$

Solution. The lighting load works at unity p.f. and, therefore, its lagging kVAR is zero. The lagging kVAR are taken by the loads (*ii*) and (*iv*), whereas loads (*iii*) and (*v*) take the leading kVAR.For station power factor to be unity, the total lagging kVAR must be neutralised by the total leadingkVAR. We know that kVAR = kW tan ϕ.

∴ Total lagging kVAR taken by loads (*ii*) and (*iv*)

$= 400 \tan (\cos^{-1} 0{\cdot}707) + 500 \tan (\cos^{-1} 0{\cdot}6)$

$= 400 \cdot 1 + 500 \cdot 1{\cdot}33 = 1065$

Leading kVAR taken by the load (*iii*)

$= 800 \tan (\cos^{-1} 0{\cdot}8) = 800 \cdot 0{\cdot}75 = 600$

∴ Leading kVAR to be taken by synchronous motor

= 1065 – 600 = 465 kVAR

Motor input = output/efficiency = 540/0·9 = 600 kWIf ϕ is the phase angle of synchronous motor, then,

$\tan \phi = \text{kVAR/kW} = 465/600 = 0{\cdot}775$

∴ $\phi = \tan^{-1} 0{\cdot}775 = 37{\cdot}77^o$

∴ p.f. of synchronous motor = cos ϕ = cos 37·77º = **0·79 leading**

Therefore, in order that the station power factor may become unity, the

Example 6.12. *An industrial load consists of* (*i*) *a synchronous motor of 100 metric h.p.* (*ii*) *induction motors aggregating 200 metric h.p., 0·707 power factor lagging and 82% efficiency and* (*iii*) *lighting load aggregating 30 kW.*

The tariff is Rs 100 per annum per kVA maximum demand plus 6 paise per kWh. Find the annual saving in cost if the synchronous motor operates at 0·8 p.f. leading, 93% efficiency instead of 0·8 p.f. lagging at 93% efficiency.

synchronous motor shouldbe operated at a p.f. of 0·79 leading.

Solution. The annual power bill will be calculated under two conditions *viz.*, (*a*) when synchro-nous motor runs with lagging p.f. and (*b*) when synchronous motor runs with a leading p.f.

(a) When synchronous motor runs at p.f. 0·8 lagging. We shall find the combined kW and then calculate total kVA maximum demand using the relation :

$$\text{kVA} = \sqrt{(\text{kW})^2 + (\text{kVAR})^2}$$

Input to synchronous motor $= \dfrac{100 \cdot 735 \cdot 5}{0 \cdot 93 \cdot 1000} = 79$ kW

*Lagging kVAR taken by the synchronous motor

$= 79 \tan(\cos^{-1} 0{\cdot}8) = 79 \cdot 0{\cdot}75 = 59{\cdot}25$ kVAR

Input to induction motors $= \dfrac{200 \cdot 735 \cdot 5}{0 \cdot 82 \cdot 1000} = 179.4$ kW

Lagging kVAR taken by induction motors

$= 179{\cdot}4 \tan(\cos^{-1} 0{\cdot}707) = 179{\cdot}4 \cdot 1 =$

179·4 kVARSince lighting load works at unity p.f., its lagging kVAR is zero.

∴ Total lagging kVAR = 59·25 + 179·4 = 238·65 kVARTotal active power = 79 + 179·4 + 30 = 288·4 kW

Total kVA = $\sqrt{(238.65)^2 + (288 \cdot 4)^2}$ = 374·4 kVA

Annual kVA demand charges = Rs 100 · 374·4 = Rs 37,440

* Since the synchronous motor in this case runs at lagging p.f., it takes lagging kVAR.

Energy consumed/year = 288·4 · 8760 = 25,26384 kWh Annual energy charges = Rs 0·06 · 25,26,384 = Rs 1,51,583

Total annual bill = Rs (37,440 + 1,51,583) = Rs 1,89,023

(b) When synchronous motor runs at p.f. 0·8 leading. As the synchronous motor runs at leading p.f. of 0·8 (instead of 0·8 p.f. lagging), therefore, it takes now 59·25 leading kVAR. The lagging kVAR taken by induction motors are the same as before *i.e.*, 179·4.

∴ Net lagging kVAR = 179·4 – 59·25 = 120·15

Total active power = Same as before *i.e.*, 288·4 kW

$\sqrt{(120.15)^2 + (288 \cdot 4)^2}$

Total kVA == 312·4

Annual kVA demand charges = Rs 100 · 312·4 = Rs 31,240 Annual energy charges = Same as before *i.e.*, Rs 1,51,583

Total annual bill = Rs (31,240 + 1,51,583) = Rs 1,82,823

∴ Annual saving = Rs (1,89,023 – 1,82,823) = **Rs 6200**

UNIT-III

POWER SYSTEM NETWORK MATRICES

The bus admittance matrix, YBUS plays a very important role in computer aided power system analysis. It can be formed in practice by either of the methods as under:

1. Rule of Inspection
2. Singular Transformation
3. Non-Singular Transformation
4. ZBUS Building Algorithms, etc.

The performance equations of a given power system can be considered in three different frames of reference as discussed below:

Frames of Reference:

Bus Frame of Reference: There are b independent equations (b = no. of buses) relating the bus vectors of currents and voltages through the bus impedance matrix and bus admittance

matrix:

EBUS = ZBUS IBUS

IBUS = YBUS EBUS

Branch Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

EBR = ZBR IBR

IBR = YBR EBR

Loop Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

ELOOP = ZLOOP ILOOP

ILOOP = YLOOP ELOOP

Of the various network matrices refered above, the bus admittance matrix (YBUS) and the bus impedance matrix (ZBUS) are determined for a given power system by the rule of inspection as explained next.

Rule of Inspection

Consider the 3-node admittance network as shown in figure5. Using the basic branch relation: I = (YV), for all the elemental currents and applying Kirchhoff‟s Current Law principle at the nodal points, we get the relations as under:

At node 1: I1 =Y1V1 + Y3 (V1-V3) + Y6 (V1 – V2)

At node 2: I2 =Y2V2 + Y5 (V2-V3) + Y6 (V2 – V1)

At node 3: 0 = Y3 (V3-V1) + Y4V3 + Y5 (V3 – V2) **(12)**

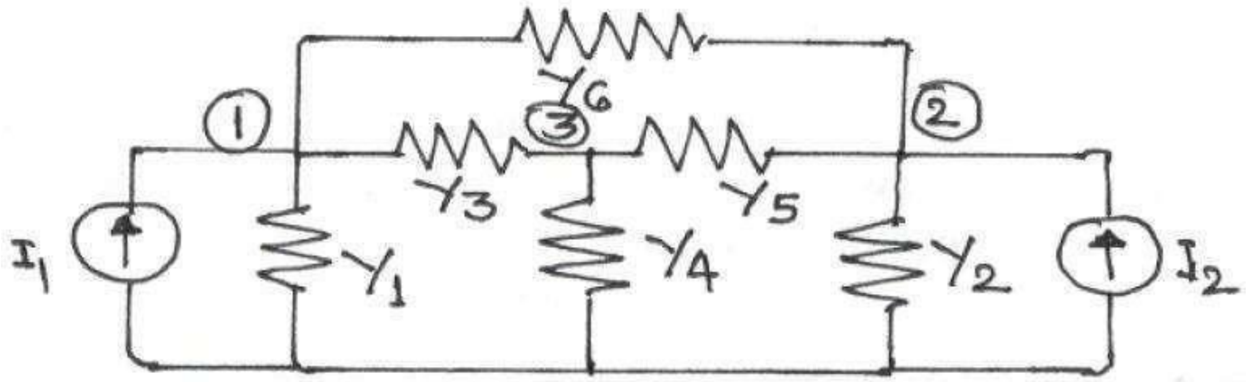


Fig. 3 Example System for finding YBUS

These are the performance equations of the given network in admittance form and they can be represented in matrix form as:

$$\begin{vmatrix} I_1 \\ I_2 \\ 0 \end{vmatrix} = \begin{vmatrix} (Y_1+Y_3+Y_6) & -Y_6 & -Y_3 \\ -Y_6 & (Y_2+Y_5+Y_6) & -Y_5 \\ -Y_3 & -Y_5 & (Y_3+Y_4+Y_5) \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix} \quad (13)$$

In other words, the relation of equation (9) can be represented in the form
IBUS = YBUS EBUS **(14)**
Where, YBUS is the bus admittance matrix, IBUS & EBUS are the bus current and bus voltage vectors respectively. By observing the elements of the bus admittance matrix, YBUS of equation (13), it is observed that the matrix elements can as well be obtained bya simple inspection of the given system diagram:
Diagonal elements: A diagonal element (Yii) of the bus admittance matrix, YBUS, is equal to the sum total of the admittance values of all the elements incident at the bus/node i,
Off Diagonal elements: An off-diagonal element (Yij) of the bus admittance matrix,YBUS, is equal to the negative of the admittance value of the connecting element present betweenthe buses I and j, if any. This is the principle of the rule of inspection. Thus the algorithmic equations for the rule of inspection are obtained as:

Yii = S yij (j = 1,2,…….n)
Yij = - yij (j = 1,2,…….n) **(15)**

For i = 1,2,….n, n = no. of buses of the given system, yij is the admittance of element connected between buses i and j and yii is the admittance of element connected between bus i and ground (reference bus).

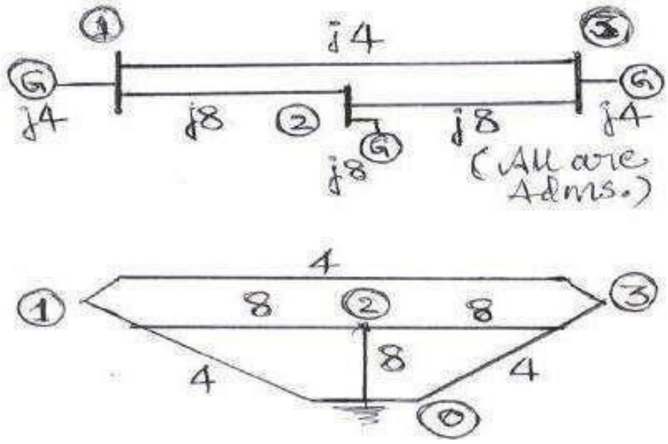
Bus impedance matrix

In cases where, the bus impedance matrix is also required, it cannot be formed by direct inspection of the given system diagram. However, the bus admittance matrix determined by the rule of inspection following the steps explained above, can be inverted to obtain the bus impedance matrix, since the two matrices are interinvertible.
Note: It is to be noted that the rule of inspection can be applied only to those power systems that do not have any mutually coupled elements.

Examples on Rule of Inspection:

Example 6: Obtain the bus admittance matrix for the admittance network shown aside by the rule of inspection

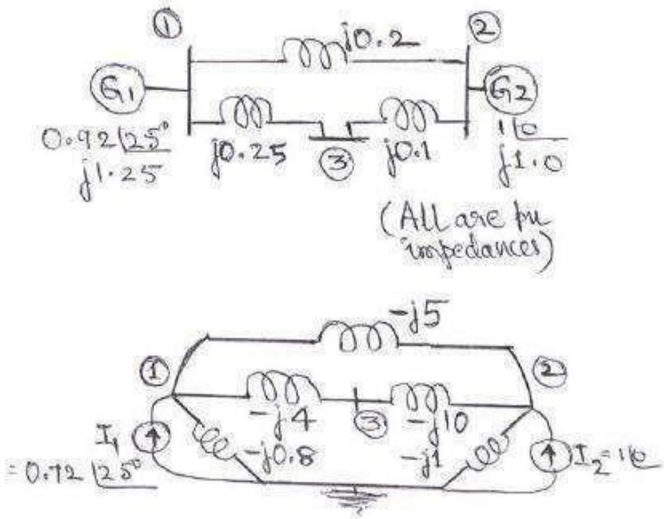
$$Y_{BUS} = j\begin{vmatrix} 16 & -8 & -4 \\ -8 & 24 & -8 \\ -4 & -8 & 16 \end{vmatrix}$$

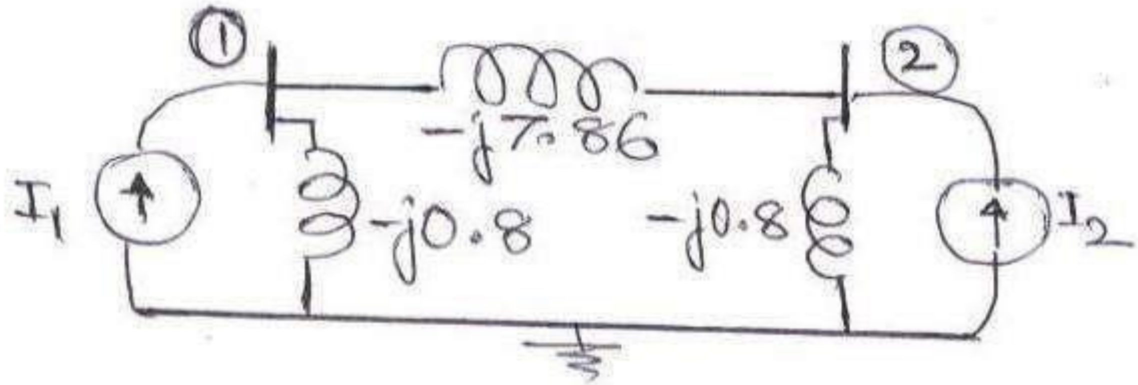


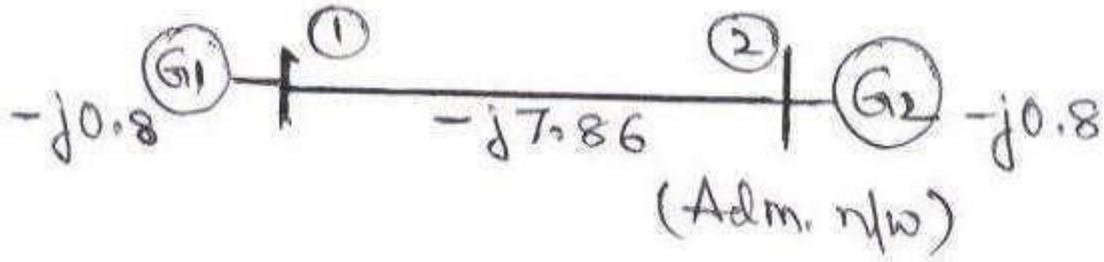
Example 7: Obtain YBUS for the impedance network shown aside by the rule of inspection. Also, determine YBUS for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.

$$Y_{BUS} = j\begin{vmatrix} -9.8 & 5 & 4 \\ 5 & -16 & 10 \\ 4 & 10 & -14 \end{vmatrix}$$

$$Z_{BUS} = Y_{BUS}^{-1}$$







$$Y_{BUS}^{New} = Y_A - Y_B Y_D^{-1} Y_C$$

$$Y_{BUS} = j\begin{vmatrix} -8.66 & 7.86 \\ 7.86 & -8.66 \end{vmatrix}$$

SINGULAR TRANSFORMATIONS

The primitive network matrices are the most basic matrices and depend purely on the impedance or admittance of the individual elements. However, they do not contain any information about the behaviour of the interconnected network variables. Hence, it is necessary to transform the primitive matrices into more meaningful matrices which can relate variables of the interconnected network.

Bus admittance matrix, YBUS and Bus impedance matrix, ZBUS

In the bus frame of reference, the performance of the interconnected network is described by *n* independent nodal equations, where *n* is the total number of buses (n+1nodes are present, out of which one of them is designated as the reference node).
For example a 5-bus system will have 5 external buses and 1 ground/ ref. bus). The performance equation relating the bus voltages to bus current injections in bus frame of reference in admittance form is given by

IBUS = YBUS EBUS **(17)**
Where EBUS = vector of bus voltages measured with respect to reference bus
IBUS = Vector of currents injected into the bus
YBUS = bus admittance matrix
The performance equation of the primitive network in admittance form is given by
i + j = [y] v
Pre-multiplying by At (transpose of A), we obtain

At i +At j = At [y] v **(18)**
However, as per equation (4),

At i =0,
since it indicates a vector whose elements are the algebraic sum of element currents incidentat a bus, which by Kirchhoff"s law is zero. Similarly, At j gives the algebraic sum of all source currents incident at each bus and this is nothing but the total current injected at the bus. Hence,

At j = IBUS **(19)**
Thus from (18) we have, IBUS = At [y] v **(20)**
However, from (5), we have
v =A EBUS
And hence substituting in (20) we get,

IBUS = At [y] A EBUS **(21)**
Comparing (21) with (17) we obtain,

YBUS = At [y] A **(22)**
The bus incidence matrix is rectangular and hence singular. Hence, (22) gives a singular transformation of the primitive admittance matrix [y]. The bus impedance matrix is given by ,

ZBUS = $YBUS^{-1}$ **(23)**
Note: This transformation can be derived using the concept of power invariance, however, since the transformations are based purely on KCL and KVL, the transformation will obviously be power invariant.

Examples on Singular Transformation:

Example 8: For the network of Fig E8, form the primitive matrices [z] & [y] and obtain the bus admittance matrix by singular transformation. Choose a Tree T(1,2,3). The data is given in Table E8.

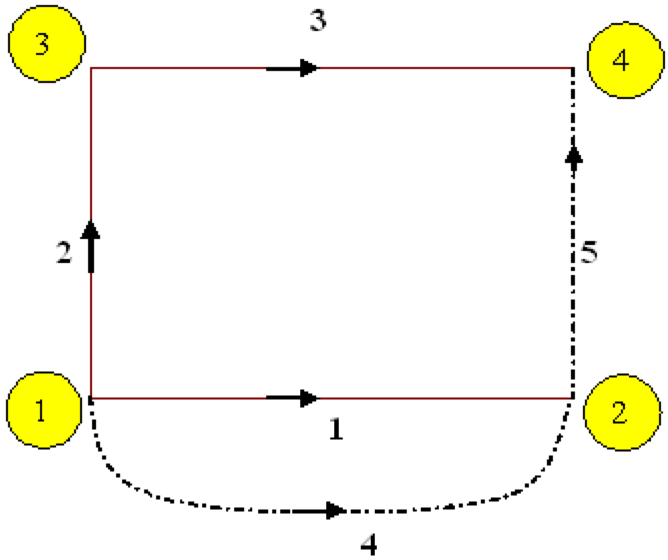


Fig E8 System for Example-8

Table E8: Data for Example-8

Elements	Self impedance	Mutual impedance
1	j 0.6	-
2	j 0.5	j 0.1(with element 1)
3	j 0.5	-
4	j 0.4	j 0.2 (with element 1)
5	j 0.2	-

Solution:

The bus incidence matrix is formed taking node 1 as the reference bus.

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

The primitive incidence matrix is given by

$$[z] = \begin{bmatrix} j0.6 & j0.1 & 0.0 & j0.2 & 0.0 \\ j0.1 & j0.5 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & j0.5 & 0.0 & 0.0 \\ j0.2 & 0.0 & 0.0 & j0.4 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & j0.2 \end{bmatrix}$$

The primitive admittance matrix [y] = [z]-1 and given by,

$$[y] = \begin{bmatrix} -j2.0833 & j0.4167 & 0.0 & j1.0417 & 0.0 \\ j0.4167 & -j2.0833 & 0.0 & -j0.2083 & 0.0 \\ 0.0 & 0.0 & -j2.0 & 0.0 & 0.0 \\ j1.0417 & -j0.2083 & 0.0 & -j3.0208 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -j5.0 \end{bmatrix}$$

The bus admittance matrix by singular transformation is obtained as

$$Y_{BUS} = A^t [y] A = \begin{bmatrix} -j8.0208 & j0.2083 & j5.0 \\ j0.2083 & -j4.0833 & j2.0 \\ j5.0 & j2.0 & -j7.0 \end{bmatrix}$$

$$Z_{BUS} = Y_{BUS}^{-1} = \begin{bmatrix} j0.2713 & j0.1264 & j0.2299 \\ j0.1264 & j0.3437 & j0.1885 \\ j0.2299 & j0.1885 & j0.3609 \end{bmatrix}$$

SUMMARY

The formulation of the mathematical model is the first step in obtaining the solution of any electrical network. The independent variables can be either currents or voltages. Correspondingly, the elements of the coefficient matrix will be impedances oradmittances.

Network equations can be formulated for solution of the network using graph theory, independent of the nature of elements. In the graph of a network, the tree-branches and links are distinctly identified. The complete information about the interconnection of the network, with the directions of the currents is contained in the bus incidence matrix.

The information on the nature of the elements which form the interconnected network is contained in the primitive impedance matrix. A primitive element can be represented in impedance form or admittance form. In the bus frame of reference, the performance of the interconnected system is described by *(n-1)* nodal equations, where *n* is the number of nodes. The bus admittance matrix and the bus impedance matrix relate the bus voltages and currents. These matrices can be obtained from the primitive impedance and admittance matrices.

FORMATION OF BUS IMPEDANCE MATRIX

NODE ELIMINATION BY MATRIX ALGEBRA

Nodes can be eliminated by the matrix manipulation of the standard node equations. However, *only those nodes at which current does not enter or leave the network can be considered for such elimination*. Such nodes can be eliminated either in one group or by taking the eligible nodes one after the other for elimination, as discussed next.

CASE-A: Simultaneous Elimination of Nodes:

Consider the performance equation of the given network in bus frame of reference in admittance form for a n-bus system, given by:

IBUS = YBUS EBUS **(1)**

Where IBUS and EBUS are n-vectors of injected bus current and bus voltages and YBUS is the square, symmetric, coefficient bus admittance matrix of order n. Now, of the n buses present in the system, let p buses be considered for node elimination so that the reduced system after elimination of p nodes would be retained with m (= n-p) nodes only. Hence the corresponding performance equation would be similar to (1) except that the coefficient matrix would be of order m now, i.e.,

IBUS = YBUS^{new} EBUS **(2)**

Where YBUS^{new} is the bus admittance matrix of the reduced network and the vectors

IBUS and EBUS are of order m. It is assumed in (1) that IBUS and EBUS are obtained with their elements arranged such that the elements associated with p nodes to be eliminated are in the lower portion of the vectors. Then the elements of YBUS also get located accordingly so that (1) after matrix partitioning yields,

$$\begin{vmatrix} I_{BUS-m} \\ I_{BUS-p} \end{vmatrix} = \begin{matrix} & m & p \\ m & \begin{vmatrix} Y_A & Y_B \\ Y_C & Y_D \end{vmatrix} \\ p & & \end{matrix} \begin{vmatrix} E_{BUS-m} \\ E_{BUS-p} \end{vmatrix} \qquad (3)$$

Where the self and mutual values of YA and YD are those identified only with the nodes to be retained and removed respectively and YC=YBt is composed of only the corresponding mutual admittance values, that are common to the nodes m and p.

Now, for the p nodes to be eliminated, it is necessary that, each element of the vector IBUS-p should be zero. Thus we have from (3):

$IBUS\text{-}m = Y_A E_{BUS-m} + Y_B E_{BUS-p}$

$IBUS\text{-}p = Y_C E_{BUS-m} + Y_D E_{BUS-p} = 0$

(4)

Solving,

$\mathbf{E_{BUS-p} = - YD^{-1} Y_C E_{BUS-m}}$

(5)

Thus, by simplification, we obtain an expression similar to (2) as,

$\mathbf{I_{BUS-m}} = \{Y_A - Y_B Y_D\text{-}1 Y_C\}\ \mathbf{E_{BUS-m}}$

(6)

Thus by comparing (2) and (6), we get an expression for the new bus admittance matrix in terms of the sub-matrices of the original bus admittance matrix as:

$\mathbf{Y_{BUSnew} = \{Y_A - Y_B Y_D\ -1 Y_C\}}$

(7)

This expression enables us to construct the given network with only the necessary nodes retained and all the unwanted nodes/buses eliminated. However, it can be observed from (7) that the expression involves finding the inverse of the sub-matrix YD (of order p). This would be computationally very tedious if p, the nodes to be eliminated is very large, especially for real practical systems. In such cases, it is more advantageous to eliminate the unwanted nodes from the given network by considering one node only at a time for elimination, as discussed next.

CASE-B: Separate Elimination of Nodes:

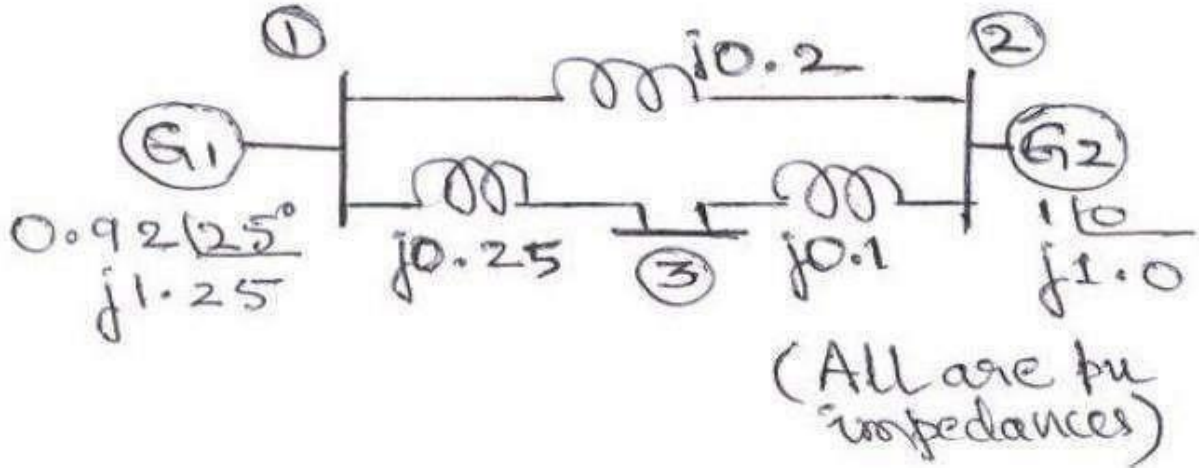
Here again, the system buses are to be renumbered, if necessary, such that the node to be removed always happens to be the last numbered one. The sub-matrix YD then would be a single element matrix and hence it inverse would be just equal to its own reciprocal value. Thus the generalized algorithmic equation for finding the elements of the new bus admittance matrix can be obtained from (6) as,

$\mathbf{Yij^{new} = Yij^{old} - Yin\ Ynj / Ynn}$ " i,j = 1,2,…… n. **(8)**

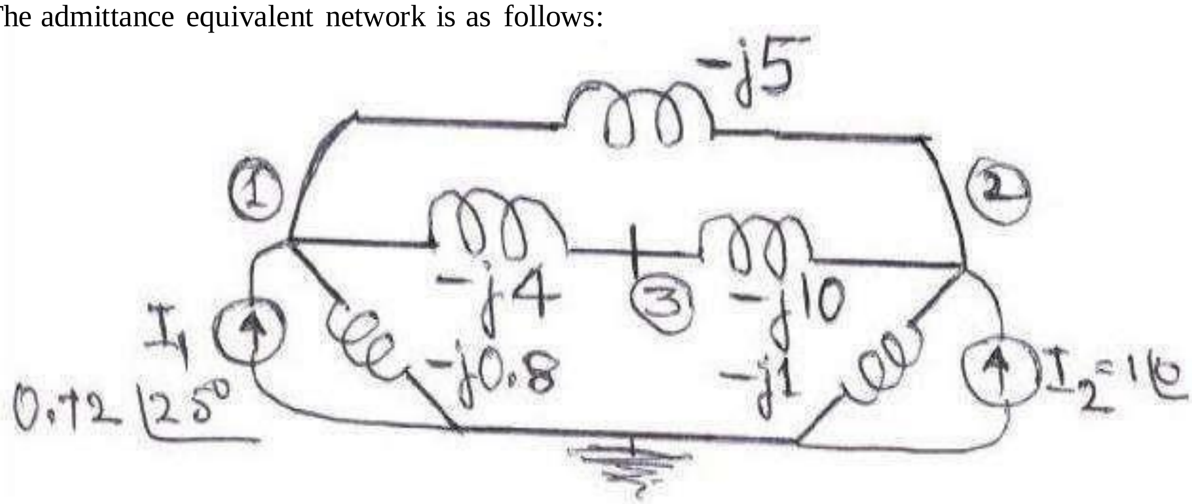
Each element of the original matrix must therefore be modified as per (7). Further, this procedure of eliminating the last numbered node from the given system of n nodes is to be iteratively repeated p times, so as to eliminate all the unnecessary p nodes from the original system.

Examples on Node elimination:

Example-1: Obtain YBUS for the impedance network shown below by the rule of inspection. Also, determine YBUS for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.



The admittance equivalent network is as follows:



The bus admittance matrix is obtained by RoI as:

$$Y_{BUS} = j\begin{vmatrix} -9.8 & 5 & 4 \\ 5 & -16 & 10 \\ 4 & 10 & -14 \end{vmatrix}$$

The reduced matrix after elimination of node 3 from the given system is determined as per the equation:

$$Y_{BUS}^{New} = Y_A - Y_B Y_D^{-1} Y_C$$

$Y_{BUS}^{new} =$

n/n	1	2
1	-j8.66	j7.86
2	j7.86	-j8.66

Alternatively,

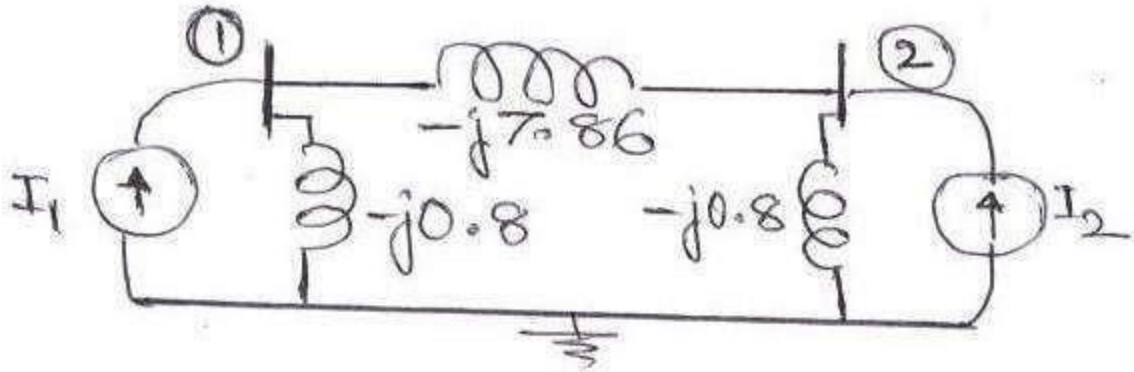
$$Y_{ij}^{new} = Y_{ij}^{old} - Y_{i3}\, Y_{3j} / Y_{33} \qquad \forall\ i,j = 1,2.$$

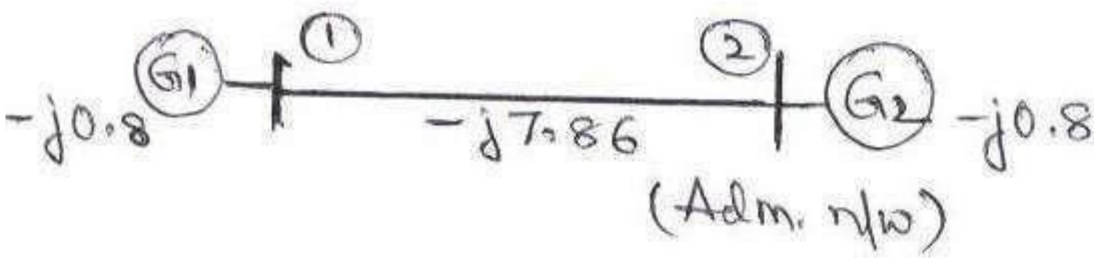
$$Y_{11} = Y_{11} - Y_{13}Y_{31}/\ Y_{33} \ = -j8.66$$

$$Y_{22} = Y_{22} - Y_{23}Y_{32}/\ Y_{33} \ = -j8.66$$

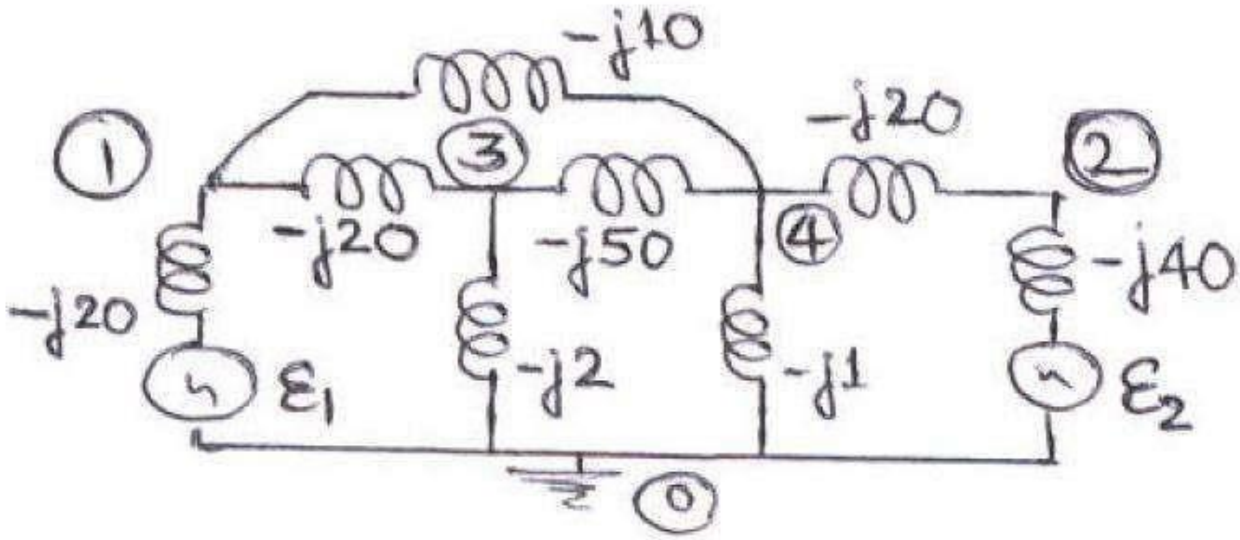
$$Y_{12} = Y_{21} = Y_{12} - Y_{13}Y_{32}/Y_{33} = j7.86$$

Thus the reduced network can be obtained again by the rule of inspection as shown below.





Example-2: Obtain YBUS for the admittance network shown below by the rule of inspection. Also, determine YBUS for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.



$Y_{BUS}=$

n/n	1	2	3	4
1	-j50	0	j20	j10
2	0	-j60	0	j72
3	j20	0	-j72	j50
4	j10	j72	j50	-j81

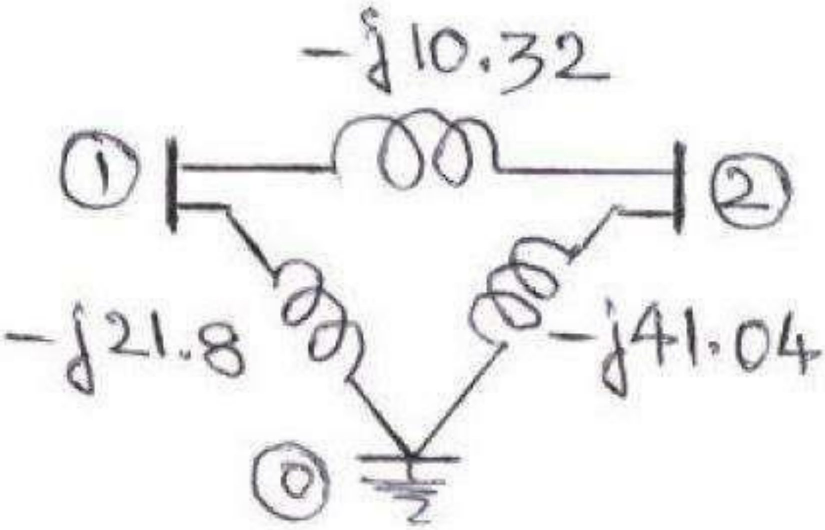
$= \begin{vmatrix} Y_A & Y_B \\ Y_C & Y_D \end{vmatrix}$

$$Y_{BUS}^{New} = Y_A - Y_B Y_D^{-1} Y_C$$

$Y_{BUS}^{new} =$

n/n	1	2
1	-j32.12	j10.32
2	j10.32	-j51.36

Thus the reduced system of two nodes can be drawn by the rule of inspection as under:


-j10.32
1
2
-j21.8
-j41.04
0

UNIT IV

FORMATION OF Z-BUS

ZBUS building

FORMATION OF BUS IMPEDANCE MATRIX

The bus impedance matrix is the inverse of the bus admittance matrix. An alternative method is possible, based on an algorithm to form the bus impedance matrix directly from system parameters and the coded bus numbers. The bus impedance matrix is formedadding one element at a time to a partial network of the given system. The performance equation of the network in bus frame of reference in impedance form using the currents as independent variables is given in matrix form by

$$\bar{E}_{bus} = [Z_{bus}]\bar{I}_{bus} \tag{9}$$

When expanded so as to refer to a *n* bus system, (9) will be of the form

$$E_1 = Z_{11}I_1 + Z_{12}I_2 + \ldots\ldots + .Z_{1k}I_k \ldots + Z_{1n}I_n$$

$$\vdots$$

$$\vdots$$

$$E_k = Z_{k1}I_1 + Z_{k2}I_2 + \ldots\ldots + Z_{kk}I_k + \ldots + Z_{kn}I_n$$

$$\vdots$$

$$\vdots$$

$$E_n = Z_{n1}I_1 + Z_{n2}I_2 + \ldots\ldots\ldots + Z_{nk}I_k + \ldots\ldots + Z_{nn}I_n \tag{10}$$

Now assume that the bus impedance matrix Zbus is known for a partial network of *m* buses and a known reference bus. Thus, Zbus of the partial network is of dimension *m*x*m*. If now a new element is added between buses *p* and *q* we have the following twopossibilities:

(i) *p* is an existing bus in the partial network and *q* is a new bus; in this case *p*-*q* is a **branch** added to the p-network as shown in Fig 1a, and

(ii) both *p* and *q* are buses existing in the partial network; in this case *p-q* is a **link** added to the p-network as shown in Fig 1b.

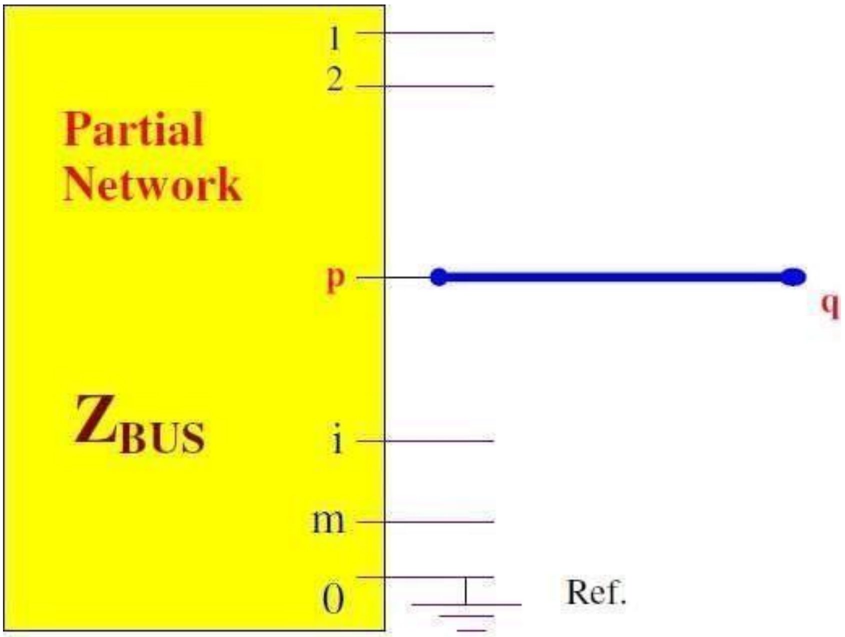


Fig 1a. Addition of branch p-q

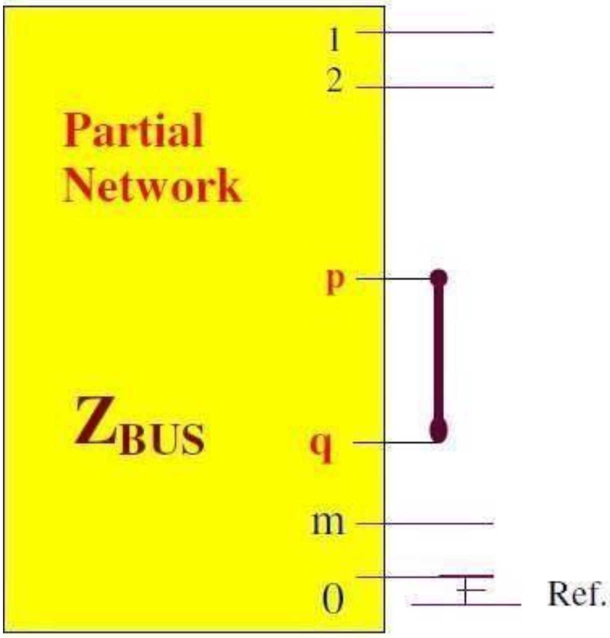


Fig 1b. Addition of link p-q

If the added element ia a branch, p-q, then the new bus impedance matrix would be of order m+1, and the analysis is confined to finding only the elements of the new row and column (corresponding to bus-q) introduced into the original matrix. If the added element ia a link, p-q, then the new bus impedance matrix will remain unaltered with regard to its order. However, all the elements of the original matrix are updated to take account of the effect of the link added.

ADDITION OF A BRANCH

Consider now the performance equation of the network in impedance form with the added branch p-q, given by

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ E_q \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\ \vdots & & & & & & \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\ \vdots & & & & & & \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\ Z_{q1} & Z_{q2} & \cdots & Z_{qp} & \cdots & Z_{qm} & Z_{qq} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_q \end{bmatrix} \quad (11)$$

It is assumed that the added branch *p-q* is mutually coupled with some elements of the partial network and since the network has bilateral passive elements only, we have

Vector ypq-rs is not equal to zero and **Zij= Zji** " i,j=1,2,…m,q
(12)

To find Zqi:

The elements of last row-q and last column-q are determined by injecting a current of 1.0 pu at the bus-i and measuring the voltage of the bus-q with respect to the reference bus-0, as shown in Fig.2. Since all other bus currents are zero, we have from (11) that

Ek = Zki Ii = Zki " k = 1, 2,…i........ p,….m, q
(13)
Hence, Eq = Zqi ; Ep = Zpi ………

Also, Eq=Ep -vpq ; so that Zqi = Zpi - vpq " i =1, 2,…i…....p,….m, _q
(14)

To find vpq:

In terms of the primitive admittances and voltages across the elements, the current through the elements is given by

$$\begin{bmatrix} i_{pq} \\ \bar{i}_{rs} \end{bmatrix} = \begin{bmatrix} y_{pq,pq} & \bar{y}_{pq,rs} \\ \bar{y}_{rs,pq} & \bar{y}_{rs,rs} \end{bmatrix} \begin{bmatrix} v_{pq} \\ \bar{v}_{rs} \end{bmatrix} \tag{15}$$

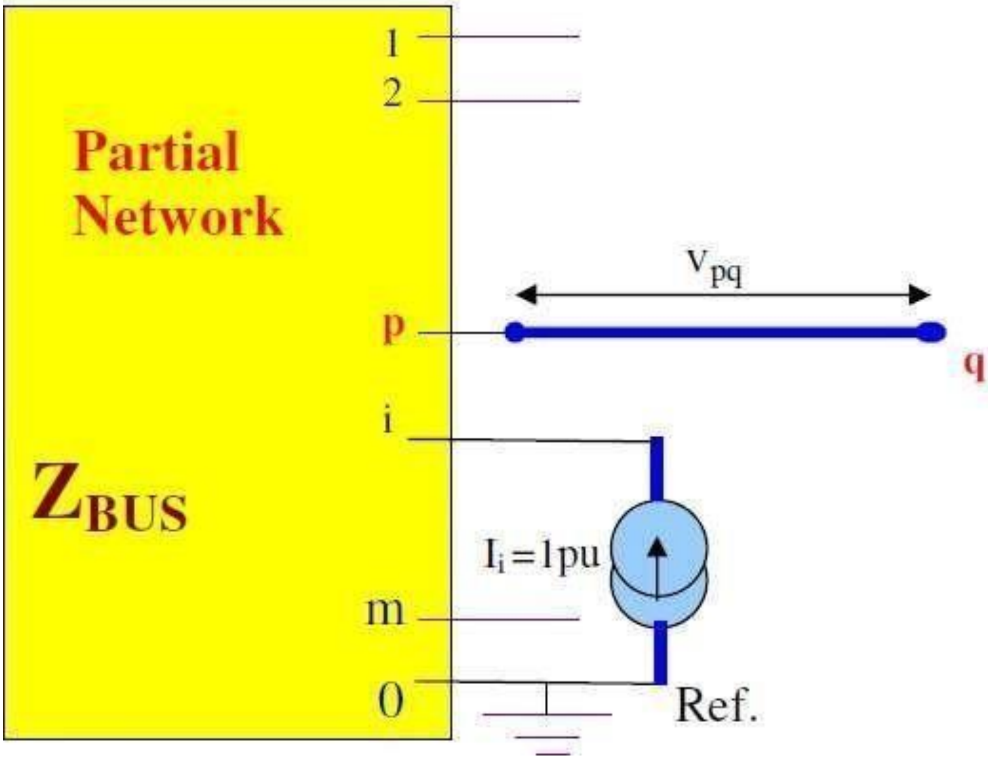


Fig.2 Calculation for Zqi

where i_{pq} is current through element *p-q*

$\bar{i}_{rs}$ is vector of currents through elements of the partial network

v_{pq} is voltage across element *p-q*

$y_{pq,pq}$ is self – admittance of the added element

$\bar{y}_{pq,rs}$ is the vector of mutual admittances between the added elements *p-q* and elements *r-s* of the partial network.

$\bar{v}_{rs}$ is vector of voltage across elements of partial network.

$\bar{y}_{rs,pq}$ is transpose of $\bar{y}_{pq,rs}$.

$\bar{y}_{rs,rs}$ is the primitive admittance of partial network.

Since the current in the added branch p-q, is zero, $i_{pq} = 0$. We thus have from (15),

$$i_{pq} = y_{pq,pq} v_{pq} + \bar{y}_{pq,rs} \bar{v}_{rs} = 0 \tag{16}$$

Solving, $v_{pq} = -\frac{\overline{y}_{pq,rs}\overline{v}_{rs}}{y_{pq,pq}}$ or

$$v_{pq} = -\frac{\overline{y}_{pq,rs}\left(\overline{E}_r - \overline{E}_s\right)}{y_{pq,pq}} \tag{17}$$

Using (13) and (17) in (14), we get

$$Z_{qi} = Z_{pi} + \frac{\overline{y}_{pq,rs}\left(\overline{Z}_{ri} - \overline{Z}_{si}\right)}{y_{pq,pq}} \quad i = 1,2\ldots\ldots m; i \neq q \tag{18}$$

To find zqq:

The element Z_{qq} can be computed by injecting a current of 1pu at bus-q, $I_q = 1.0$ pu. As before, we have the relations as under:

$E_k = Z_{kq} I_q = Z_{kq}$ $\quad \forall\ k = 1, 2,\ldots i\ldots\ldots p,\ldots m, q$ (19)

Hence, $E_q = Z_{qq}$; $E_p = Z_{pq}$; Also, $E_q = E_p - v_{pq}$; so that $Z_{qq} = Z_{pq} - v_{pq}$ (20)

Since now the current in the added element is $i_{pq} = -I_q = -1.0$, we have from (15)

$$i_{pq} = y_{pq,pq}v_{pq} + \overline{y}_{pq,rs}\overline{v}_{rs} = -1$$

Solving, $v_{pq} = -1 + \frac{\overline{y}_{pq,rs}\overline{v}_{rs}}{y_{pq,pq}}$

$$v_{pq} = -1 + \frac{\overline{y}_{pq,rs}\left(\overline{E}_r - \overline{E}_s\right)}{y_{pq,pq}} \tag{21}$$

Using (19) and (21) in (20), we get

$$Z_{qq} = Z_{pq} + \frac{1 + \overline{y}_{pq,rs}\left(\overline{Z}_{rq} - \overline{Z}_{sq}\right)}{y_{pq,pq}} \tag{22}$$

Special Cases

The following special cases of analysis concerning ZBUS building can be considered with respect to the addition of branch to a p-network.

Case (a): If there is no mutual coupling then elements of $\bar{y}_{pq,rs}$ are zero. Further, if p is the reference node, then $E_p=0$. thus,

$$Z_{pi} = 0 \qquad i = 1,2......m : i \neq q$$

And $$Z_{pq} = 0.$$

Hence, from (18) (22) $$Z_{qi} = 0 \qquad i = 1,2......m; i \neq q$$

And $$Z_{qq} = z_{pq,pq} \qquad \backslash \quad (23)$$

Case (b): If there is no mutual coupling and if p is not the ref. bus, then, from (18) and (22), we again have,

$$Z_{qi} = Z_{pi}, \quad i = 1,2....m; i \neq q$$

$$Z_{qq} = Z_{pq} + z_{pq,pq} \qquad (24)$$

ADDITION OF A LINK

Consider now the performance equation of the network in impedance form with the added link p-l, (p-l being a fictitious branch and l being a fictitious node) given by

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ E_l \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\ \vdots & & & & & & \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\ \vdots & & & & & & \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\ Z_{l1} & Z_{l2} & \cdots & Z_{li} & \cdots & Z_{lm} & Z_{ll} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_l \end{bmatrix} \qquad (25)$$

It is assumed that the added branch *p-q* is mutually coupled with some elements of the partial network and since the network has bilateral passive elements only, we have

Vector y_{pq-rs} is not equal to zero and $Z_{ij} = Z_{ji} \quad \forall\ i,j=1,2,\ldots m,l.$ (26)

To find Zli:

The elements of last row-l and last column-l are determined by injecting a current of 1.0 pu at the bus-i and measuring the voltage of the bus-q with respect to the reference bus-0, as shown in Fig.3. Further, the current in the added element is made zero by connecting a voltage source, el in series with element p-q, as shown. Since all other bus currents are zero, we have from (25) that

$$E_k = Z_{ki}\, I_i = Z_{ki} \qquad \forall\ k = 1, 2,\ldots i\ldots\ldots p,\ldots.m, l \tag{27}$$

Hence, $e_l = E_l = Z_{li}$; $E_p = Z_{pi}$; $E_p = Z_{pi}$ ………

Also, $e_l = E_p - E_q - v_{pq}$;

So that $Z_{li} = Z_{pi} - Z_{qi} - v_{pq} \quad \forall\ i=1,2,\ldots i\ldots.p,\ldots q,\ldots.m, \neq l$ (28)

To find vpq:

In terms of the primitive admittances and voltages across the elements, the current through the elements is given by

$$\begin{bmatrix} i_{pl} \\ \bar{i}_{rs} \end{bmatrix} = \begin{bmatrix} y_{pl,pl} & \bar{y}_{pl,rs} \\ \bar{y}_{rs,pl} & \bar{y}_{rs,rs} \end{bmatrix} \begin{bmatrix} v_{pl} \\ \bar{v}_{rs} \end{bmatrix} \tag{29}$$

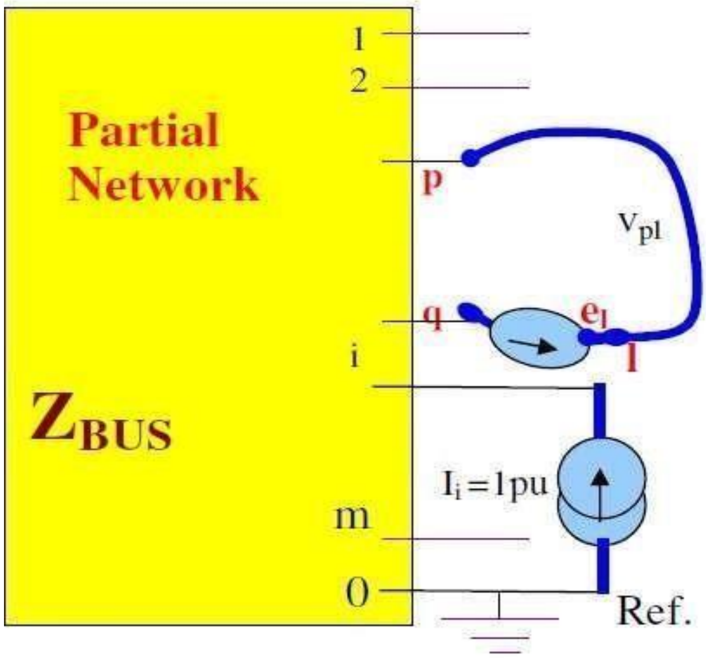


Fig.3 Calculation for Z_{li}

where i_{pl} is current through element *p-q*

$\bar{i}_{rs}$ is vector of currents through elements of the partial network

v_{pl} is voltage across element *p-q*

$y_{pl,pl}$ is self – admittance of the added element

$\bar{y}_{pl,rs}$ is the vector of mutual admittances between the added elements *p-q* and elements *r-s* of the partial network.

$\bar{v}_{rs}$ is vector of voltage across elements of partial network.

$\bar{y}_{rs,pl}$ is transpose of $\bar{y}_{pl,rs}$.

$\bar{y}_{rs,rs}$ is the primitive admittance of partial network.

Since the current in the added branch p-l, is zero, $i_{pl} = 0$. We thus have from (29),

$$i_{pl} = y_{pl,pl} v_{pl} + \bar{y}_{pl,rs} \bar{v}_{rs} = 0 \tag{30}$$

Solving, $v_{pl} = -\dfrac{\bar{y}_{pl,rs}\bar{v}_{rs}}{y_{pl,pl}}$ or

$$v_{pl} = -\frac{\bar{y}_{pl,rs}\left(\bar{E}_r - \bar{E}_s\right)}{y_{pl,pl}} \tag{31}$$

However,

$$\bar{y}_{pl,rs} = \bar{y}_{pq,rs}$$

And $$y_{pl,pl} = y_{pq,pq} \tag{32}$$

Using (27), (31) and (32) in (28), we get

$$Z_{li} = Z_{pi} - Z_{qi} + \frac{\bar{y}_{pq,rs}\left(\bar{Z}_{ri} - \bar{Z}_{si}\right)}{y_{pq,pq}} \qquad i = 1,2......m; i \neq l \tag{33}$$

To find Z_{ll}:

The element Z_{ll} can be computed by injecting a current of 1pu at bus-l, I_l = 1.0 pu. As before, we have the relations as under:

$E_k = Z_{kl} I_l = Z_{kl} \quad \forall\ k = 1, 2,\ldots i\ldots\ldots p,\ldots q,\ldots.m, l$ (34)

Hence, $e_l = E_l = Z_{ll}$; $E_p = Z_{pl}$;

Also, $e_l = E_p - E_q - v_{pl}$;

So that $Z_{ll} = Z_{pl} - Z_{ql} - v_{pl} \quad \forall\ i=1,2,\ldots i\ldots.p,\ldots q,\ldots.m, \neq l$ (35)

Since now the current in the added element is $i_{pl} = -I_l = -1.0$, we have from (29)

$$i_{pl} = y_{pl,pl} v_{pl} + \bar{y}_{pl,rs} \bar{v}_{rs} = -1$$

Solving, $v_{pl} = -1 + \dfrac{\bar{y}_{pl,rs} \bar{v}_{rs}}{y_{pl,pl}}$

$$v_{pl} = -1 + \frac{\bar{y}_{pl,rs}\left(\bar{E}_r - \bar{E}_s\right)}{y_{pl,pl}} \quad (36)$$

However,

$$\bar{y}_{pl,rs} = \bar{y}_{pq,rs}$$

And $$y_{pl,pl} = y_{pq,pq} \quad (37)$$

Using (34), (36) and (37) in (35), we get

$$Z_{ll} = Z_{pl} - Z_{ql} + \frac{1 + \bar{y}_{pq,rs}\left(\bar{Z}_{rl} - \bar{Z}_{sl}\right)}{y_{pq,pq}} \quad (38)$$

Special Cases Contd….

The following special cases of analysis concerning Z_{BUS} building can be considered with respect to the addition of link to a p-network.

Case (c): If there is no mutual coupling, then elements of $\bar{y}_{pq,rs}$ are zero. Further, if *p* is the reference node, then E_p=0. thus,

$$Z_{li} = -Z_{qi}, \quad i = 1,2\ldots.m; i \neq l$$

$$Z_{ll} = -Z_{ql} + z_{pq,pq} \quad (39)$$

From (39), it is thus observed that, when a link is added to a ref. bus, then the situation is similar to adding a branch to a fictitious bus and hence the following steps are followed:

1. The element is added similar to addition of a branch (case-b) to obtain the new matrix of order m+1.
2. The extra fictitious node, l is eliminated using the node elimination algorithm.

Case (d): If there is no mutual coupling, then elements of *pq rs y* , are zero. Further, if *p* is not the reference node, then

$$Z_{li} = Z_{pi} - Z_{qi}$$

$$\begin{aligned} Z_{ll} &= Z_{pl} - Z_{ql} - z_{pq,pq} \\ &= Z_{pp} + Z_{qq} - 2\,Z_{pq} + z_{pq,pq} \end{aligned} \tag{40}$$

MODIFICATION OF ZBUS FOR NETWORK CHANGES

An element which is not coupled to any other element can be removed easily. The Zbus is modified as explained in sections above, by adding in parallel with the element (to be removed), a link whose impedance is equal to the negative of the impedance of the element to be removed. Similarly, the impedance value of an element which is not coupledto any other element can be changed easily. The Zbus is modified again as explained in sections above, by adding in parallel with the element (whose impedance is to be changed), a link element of impedance value chosen such that the parallel equivalent impedance is equal to the desired value of impedance. When mutually coupled elements are removed, the Zbus is modified by introducing appropriate changes in the bus currents of the original network to reflect the changes introduced due to the removal of theelements.

Examples on ZBUS building

Example 1: For the positive sequence network data shown in table below, obtain ZBUS by building procedure.

Sl. No.	p-q (nodes)	Pos. seq. reactance in pu
1	0-1	0.25
2	0-3	0.20
3	1-2	0.08
4	2-3	0.06

Solution:
The given network is as shown below with the data marked on it. Assume the elements to be added as per the given sequence: 0-1, 0-3, 1-2, and 2-3.

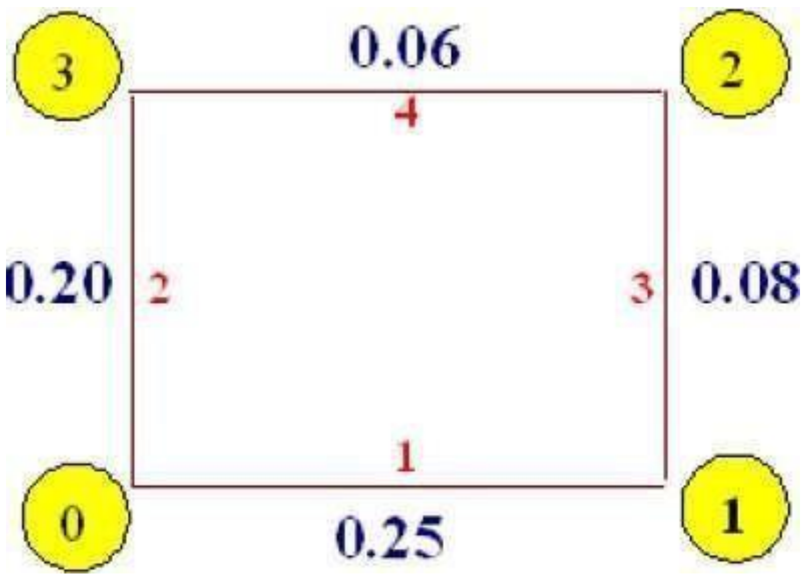
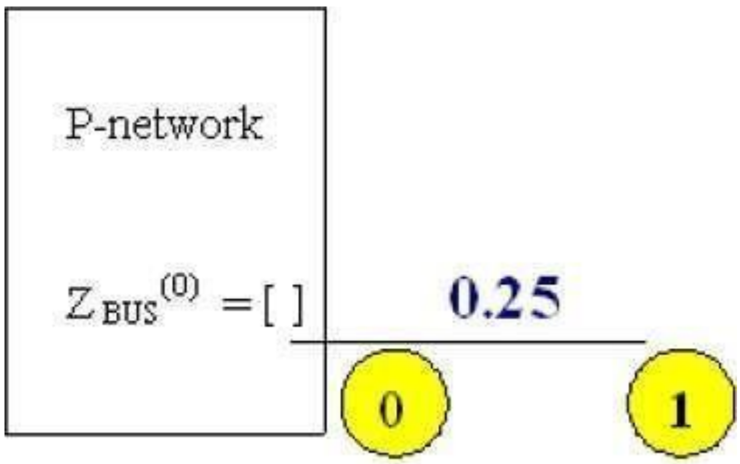


Fig. E1: Example System

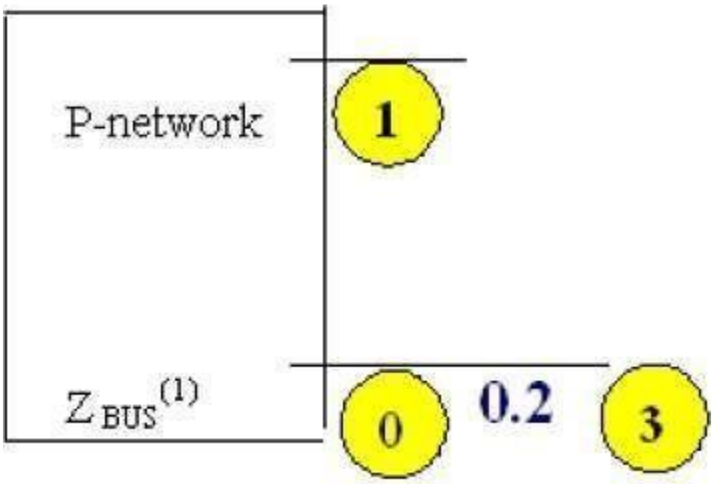
Consider building ZBUS as per the various stages of building through the consideration of the corresponding partial networks as under:

Step-1: Add element–1 of impedance 0.25 pu from the external node-1 (q=1) to internal ref. node-0 (p=0). (Case-a), as shown in the partial network;



$$Z_{BUS}^{(1)} = \begin{array}{c|c|} & 1 \\ \hline 1 & 0.25 \\ \hline \end{array}$$

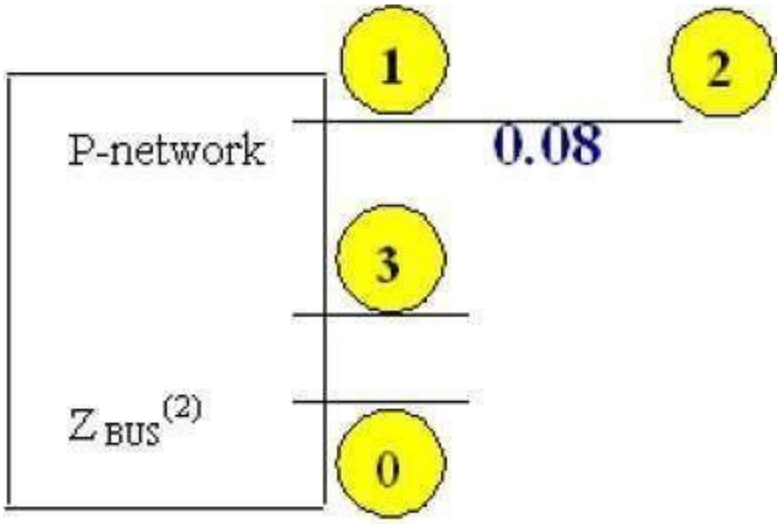
Step-2: Add element–2 of impedance 0.2 pu from the external node-3 (q=3) to internal ref. node-0 (p=0). (Case-a), as shown in the partial network;



$Z_{BUS}^{(2)} =$

	1	3
1	0.25	0
3	0	0.2

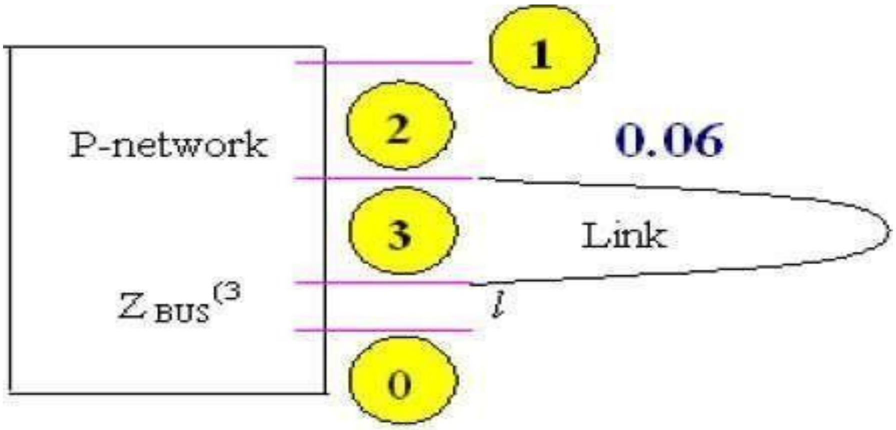
Step-3: Add element–3 of impedance 0.08 pu from the external node-2 (q=2) to internal node-1 (p=1). (Case-b), as shown in the partial network;



$Z_{BUS}^{(3)} =$

	1	3	2
1	0.25	0	0.25
3	0	0.2	0
2	0.25	0	0.33

Step-4: Add element–4 of impedance 0.06 pu between the two internal nodes, node-2 (p=2) to node-3 (q=3). (Case-d), as shown in the partial network;



$Z_{BUS}^{(4)}$ =

	1	3	2	l
1	0.25	0	0.25	0.25
3	0	0.2	0	-0.2
2	0.25	0	0.33	0.33
l	0.25	-0.2	0.33	0.59

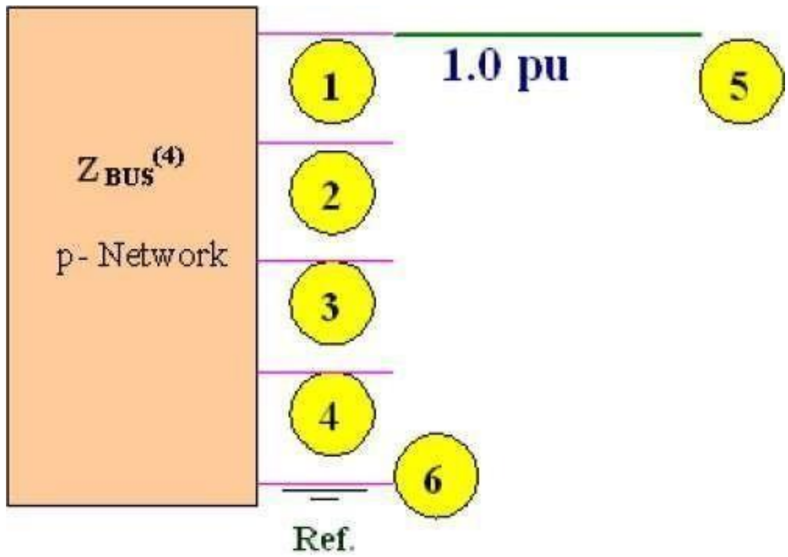
The fictitious node *l* is eliminated further to arrive at the final impedance matrix as under:

$Z_{BUS}^{(final)}$ =

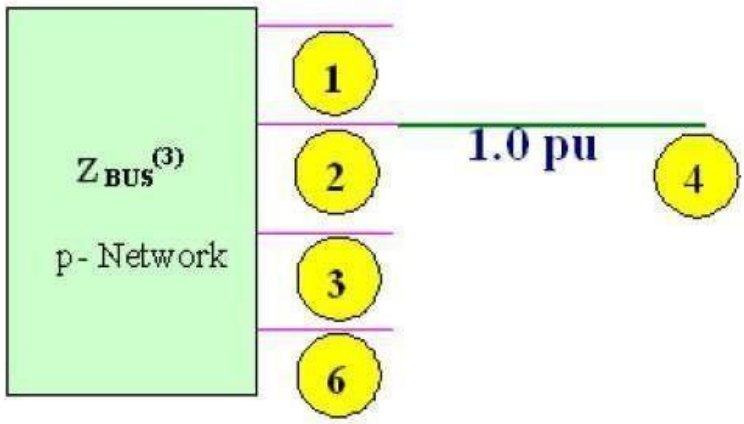
	1	3	2
1	0.1441	0.0847	0.1100
3	0.0847	0.1322	0.1120
2	0.1100	0.1120	0.1454

Z_{BUS} =

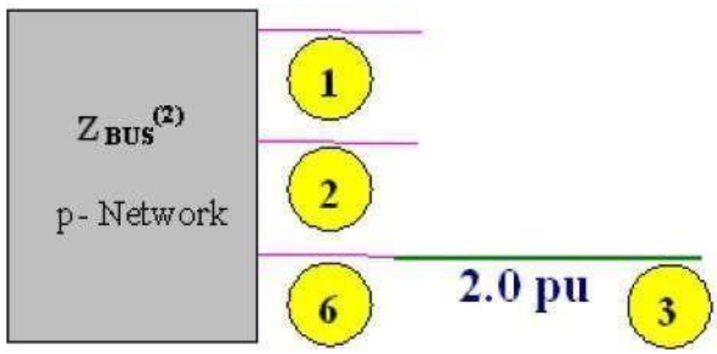
	1	2	3	4	5
1	2	0	0	0	2
2	0	2	0	2	0
3	0	0	2	0	0
4	0	2	0	3	0
5	2	0	0	0	3



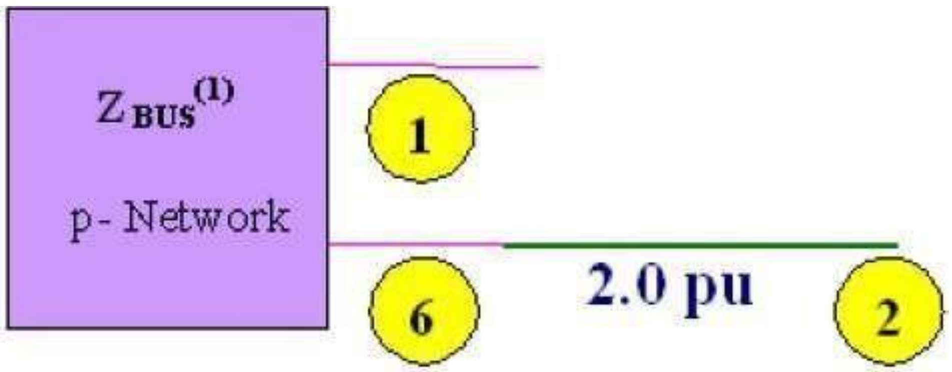
Z BUS(4)
p- Network
1
2
3
4
1.0 pu
5
6
Ref.

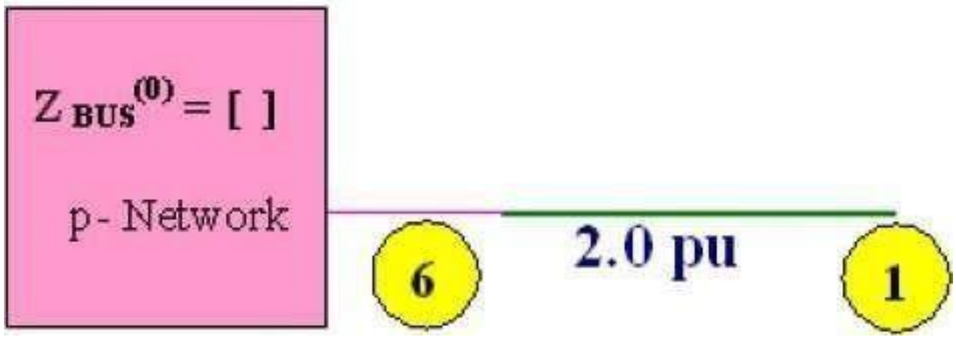


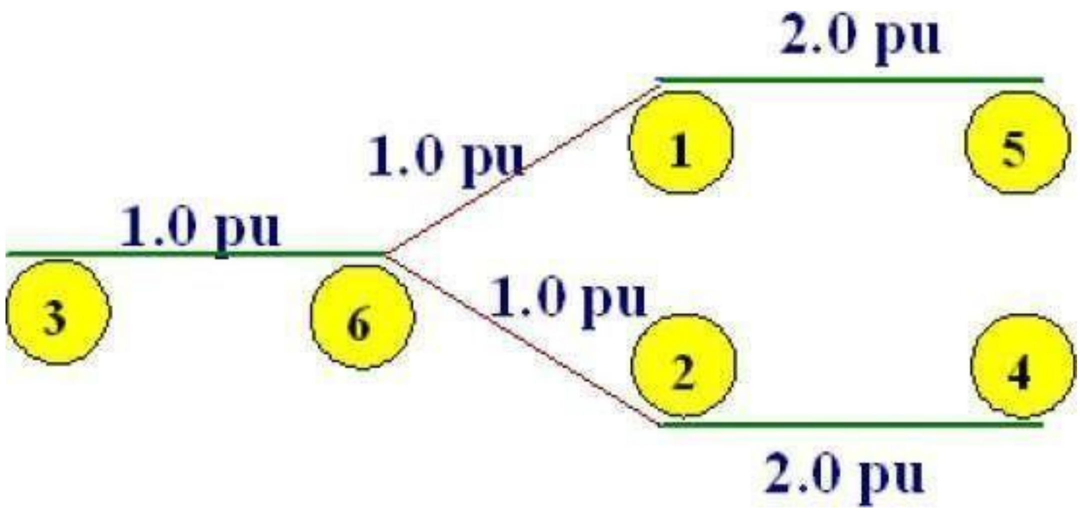
Z BUS(3)
p- Network
1
2
3
6
1.0 pu
4

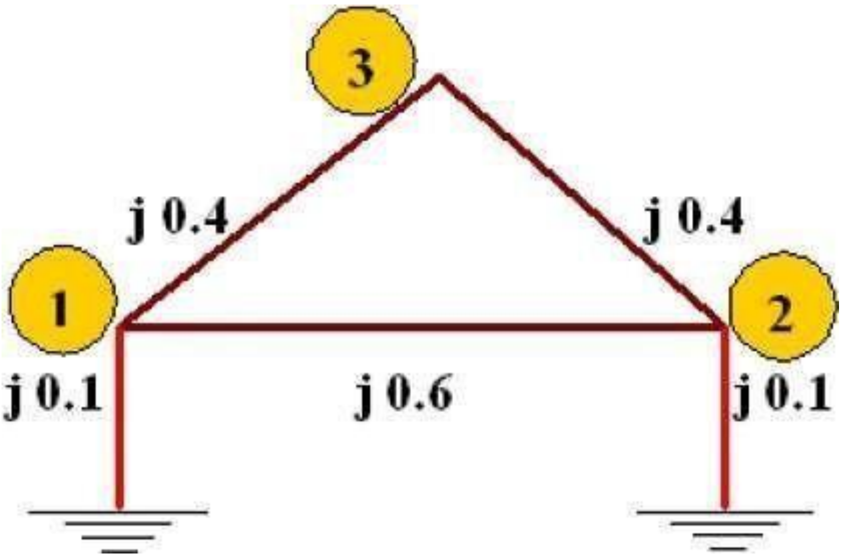


Z BUS(2)
p- Network
1
2
6
2.0 pu
3


$Z_{BUS}^{(1)}$
p- Network
1
6
2.0 pu
2

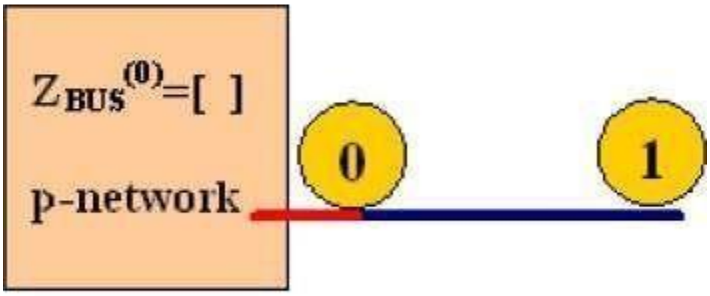

$Z_{BUS}^{(0)}$ = [ ]
p- Network
6
2.0 pu
1


2.0 pu
1.0 pu
1
5
1.0 pu
3
6
1.0 pu
2
4
2.0 pu



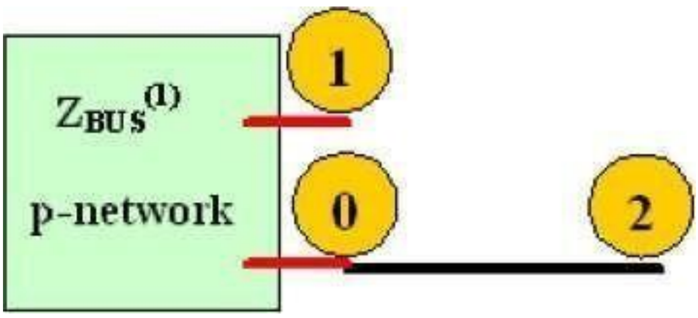
Solution: The specified system is considered with the reference node denoted by node-0. By its inspection, we can obtain the bus impedance matrix by building procedure by following the steps through the p-networks as under:

Step1: Add branch 1 between node 1 and reference node. (q =1, p = 0)



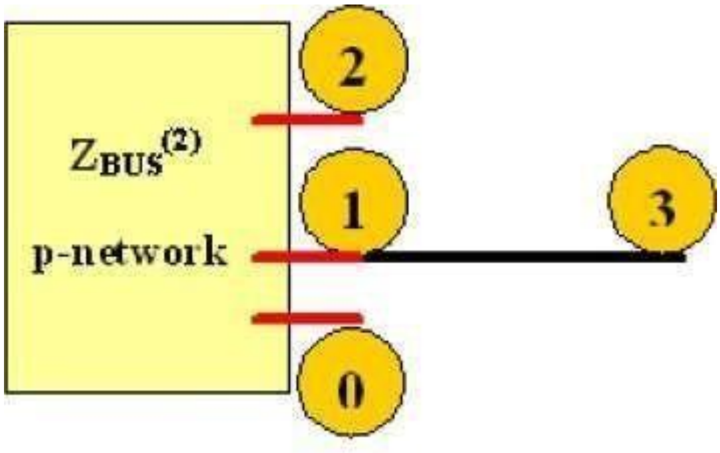
$$Z_{bus}^{(1)} = \begin{matrix} & 1 \\ 1 & [j0.1] \end{matrix}$$

Step2: Add branch 2, between node 2 and reference node. (q = 2, p = 0).



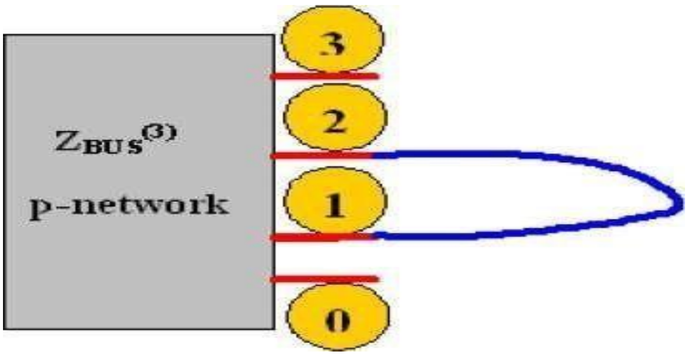
$$
Z_{Bus} = \begin{array}{c} \\ 1 \\ 2 \end{array}
\begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} j0.1 & 0 \\ 0 & j0.15 \end{bmatrix} \end{array}
$$

Step3: Add branch 3, between node 1 and node 3 (p = 1, q = 3)



$$
Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}
\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} j0.1 & 0 & j0.1 \\ 0 & j0.15 & 0 \\ j0.1 & 0 & j0.5 \end{bmatrix} \end{array}
$$

Step 4: Add element 4, which is a link between node 1 and node 2. (p = 1, q = 2)



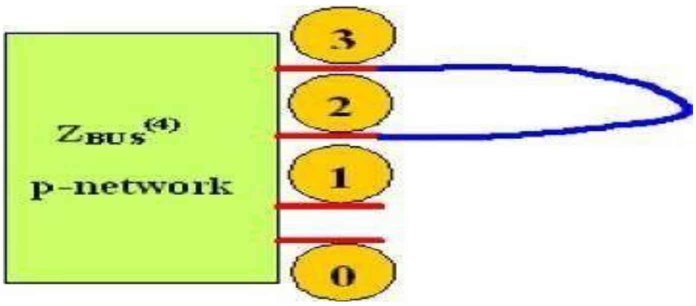
$$
Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ l \end{array}
\begin{array}{c} \begin{array}{cccc} \quad 1 \quad & \quad 2 \quad & \quad 3 \quad & \quad l \quad \end{array} \\
\begin{bmatrix} j0.1 & 0 & j0.1 & j0.1 \\ 0 & j0.15 & 0 & -j0.15 \\ j0.1 & 0 & j0.5 & j0.1 \\ j0.1 & -j0.15 & j0.1 & j0.85 \end{bmatrix} \end{array}
$$

Now the extra node-*l* has to be eliminated to obtain the new matrix of step-4, using the algorithmic relation:

$$
\mathbf{Y_{ij}^{new} = Y_{ij}^{old} - Y_{in}\, Y_{nj} \,/\, Y_{nn}} \qquad \forall\ i,j = 1,2,3.
$$

$$
Z_{bus} = \begin{array}{c} \begin{array}{ccc} \quad 1 \quad & \quad 2 \quad & \quad 3 \quad \end{array} \\
\begin{bmatrix} j0.08823 & j0.01765 & j0.08823 \\ j0.01765 & j0.12353 & j0.01765 \\ j0.08823 & j0.01765 & j0.48823 \end{bmatrix} \end{array}
$$

Step 5: Add link between node 2 and node 3 (p = 2, q=3)



$$Z_{l1} = Z_{21} - Z_{31} = j0.01765 - j0.08823 = -j0.07058$$
$$Z_{l2} = Z_{22} - Z_{32} = j0.12353 - j0.01765 = j0.10588$$
$$Z_{l3} = Z_{23} - Z_{33} = j0.01765 - j0.48823 = -j0.47058$$
$$Z_{ll} = Z_{2l} - Z_{3l} + z_{23,23}$$
$$= j0.10588 - (-j0.47058) + j0.4 = j0.97646$$

Thus, the new matrix is as under:

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ l \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 1 \end{array} \\ \begin{bmatrix} j0.08823 & j0.01765 & j0.08823 & -j0.07058 \\ j0.01765 & j0.12353 & j0.01765 & j0.10588 \\ j0.08823 & j0.01765 & j0.48823 & -j0.47058 \\ -j0.07058 & j0.10588 & -j0.47058 & j0.97646 \end{bmatrix} \end{array}$$

Node l is eliminated as shown in the previous step:

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} j0.08313 & j0.02530 & j0.05421 \\ j0.02530 & j0.11205 & j0.06868 \\ j0.05421 & j0.06868 & j0.26145 \end{bmatrix} \end{array}$$

Further, the bus admittance matrix can be obtained by inverting the bus impedance matrix as under:

$$Y_{bus} = [Z_{bus}]^{-1} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} -j14.1667 & j1.6667 & j2.5 \\ j1.6667 & -j10.8334 & j2.5 \\ j2.5 & j2.5 & -j5.0 \end{bmatrix} \end{array}$$

As a check, it can be observed that the bus admittance matrix, Y_{BUS} can also be obtained by the rule of inspection to arrive at the same answer.

UNIT-V

LOAD FLOW STUDIES

REVIEW OF NUMERICAL SOLUTION OF EQUATIONS

The numerical analysis involving the solution of algebraic simultaneous equations forms the basis for solution of the performance equations in computer aided electrical power system analyses, such as during linear graph analysis, load flow analysis (nonlinear equations), transient stability studies (differential equations), etc. Hence, it is necessary to review the general forms of the various solution methods with respect to all forms of equations, as under:

1. Solution Linear equations:
* **Direct methods**:
- Cramer‟s (Determinant) Method,
- Gauss Elimination Method (only for smaller systems),
- LU Factorization (more preferred method), etc.

* **Iterative methods**:
- Gauss Method
- Gauss-Siedel Method (for diagonally dominant systems)

3. **Solution of Nonlinear equations:**

Iterative methods only:
- Gauss-Siedel Method (for smaller systems)
- Newton-Raphson Method (if corrections for variables are small)

4. **Solution of differential equations:**

Iterative methods only:
- Euler and Modified Euler method,
- RK IV-order method,
- Milne‟s predictor-corrector method, etc.

It is to be observed that the nonlinear and differential equations can be solved only by the iterative methods. The iterative methods are characterized by the various performance features as under:

_ Selection of initial solution/ estimates
_ Determination of fresh/ new estimates during each iteration
_ Selection of number of iterations as per tolerance limit
_ Time per iteration and total time of solution as per the solution method selected
_ Convergence and divergence criteria of the iterative solution
_ Choice of the Acceleration factor of convergence, etc.

A comparison of the above solution methods is as under:
In general, the direct methods yield exact or accurate solutions. However, they are suited for only the smaller systems, since otherwise, in large systems, the possible round-off errors make the solution process inaccurate. The iterative methods are more useful when the diagonal elements of the coefficient matrix are large in comparison with the offdiagonal elements. The round-off errors in these methods are corrected at the successive steps of the iterative process.The Newton-Raphson method is very much useful for solution of non – linear equations, if all the values of the corrections for the unknowns are very small in magnitude and the initial values of unknowns are selected to be reasonably closer to the exact solution.

LOAD FLOW STUDIES

Introduction: Load flow studies are important in planning and designing future expansion of power systems. The study gives steady state solutions of the voltages at all the buses, for a particular load condition. Different steady state solutions can be obtained, for different operating conditions, to help in planning, design and operation of the power system. Generally, load flow studies are limited to the transmission system, which involves bulk power transmission. The load at the buses is assumed to be known. Load flow studies throw light on some of the important aspects of the system operation, such as: violation of voltage magnitudes at the buses, overloading of lines, overloading of generators, stability margin reduction, indicated by power angle differences between buseslinked by a line, effect of contingencies like line voltages, emergency shutdown of generators, etc. Load flow studies are required for deciding the economic operation of the power system. They are also required in transient stability studies. Hence, load flow studies play a vital role in power system studies. Thus the load flow problem consists of finding the power flows (real and reactive) and voltages of a network for given bus conditions. At each bus, there are four quantities of interest to be known for further analysis: the real and reactive power, the voltage magnitude and its phase angle. Because of the nonlinearity of the algebraic equations, describing the given power system, their solutions are obviously, based on the iterative methods only. The constraints placed on the load flow solutions could be:
_ The Kirchhoff‟s relations holding good,
_ Capability limits of reactive power sources,
_ Tap-setting range of tap-changing transformers,
_ Specified power interchange between interconnected systems,
_ Selection of initial values, acceleration factor, convergence limit, etc.

Classification of buses for LFA: Different types of buses are present based on the specified and unspecified variables at a given bus as presented in the table below:

Table 1. Classification of buses for LFA

Sl. No.	Bus Types	Specified Variables	Unspecified variables	Remarks
1	Slack/ Swing Bus	$\|V\|$, δ	P_G, Q_G	$\|V\|$, δ: are assumed if not specified as 1.0 and 0^0
2	Generator/ Machine/ PV Bus	P_G, $\|V\|$	Q_G, δ	A generator is present at the machine bus
3	Load/ PQ Bus	P_G, Q_G	$\|V\|$, δ	About 80% buses are of PQ type
4	Voltage Controlled Bus	P_G, Q_G, $\|V\|$	δ, a	'a' is the % tap change in tap-changing transformer

Importance of swing bus: The slack or swing bus is usually a PV-bus with the largest capacity generator of the given system connected to it. The generator at the swing bus supplies the power difference between the "specified power into the system at the other buses" and the "total system output plus losses". Thus swing bus is needed to supply the additional real and reactive power to meet the losses. Both the magnitude and phase angle of voltage are specified at the swing bus, or otherwise, they are assumed to be equal to 1.0 p.u. and 00 , as per flat-start procedure of iterative
solutions. The real and reactive powers at the swing bus are found by the computer routine as part of the load flow solution process. It is to be noted that the source at the swing bus is a perfect one, called the swing machine, or slack machine. It is voltage regulated, i.e., the magnitude of voltage fixed. The phase angle is the system reference phase and hence is fixed. The generator at the swing bus has a torque angle and excitation which vary or swing as the demand changes. This variation is such as to produce fixed voltage.

Importance of YBUS based LFA:

The majority of load flow programs employ methods using the bus admittance matrix, as this method is found to be more economical. The bus admittance matrix plays a very important role in load flow analysis. It is a complex, square and symmetric matrix and hence only n(n+1)/2 elements of YBUS need to be stored for a n-bus system. Further, in the YBUS matrix, Yij = 0, if an incident element is not present in the system connectingthe buses „i" and „j". since in a large power system, each bus is connected only to a fewer buses through an incident element, (about 6-8), the coefficient matrix, YBUS of such systems would be highly sparse, i.e., it will have many zero valued elements in it. This is defined by the sparsity of the matrix, as under:

$$\text{Percentage sparsity of a given matrix of } n^{th} \text{ order:} = \frac{\text{Total no. of zero valued elements of } Y_{BUS}}{\text{Total no. of entries of } Y_{BUS}}$$

$$S = (Z / n^2) \times 100 \ \% \qquad (1)$$

The percentage sparsity of Y_{BUS}, in practice, could be as high as 80-90%, especially for very large, practical power systems. This sparsity feature of YBUS is extensively used in reducing the load flow calculations and in minimizing the memory required to store the coefficient matrices. This is due to the fact that only the non-zero elements YBUS can be stored during the computer based implementation of the schemes, by adopting the suitable optimal storage schemes. While YBUS is thus highly sparse, it‟s inverse, ZBUS, the bus impedance matrix is not so. It is a FULL matrix, unless the optimal bus ordering schemes are followed before proceeding for load flow analysis.

THE LOAD FLOW PROBLEM

Here, the analysis is restricted to a balanced three-phase power system, so that the analysis can be carried out on a single phase basis. The per unit quantities are used for allquantities. The first step in the analysis is the formulation of suitable equations for the power flows in the system. The power system is a large interconnected system, where various buses are connected by transmission lines. At any bus, complex power is injected into the bus by the generators and complex power is drawn by the loads. Of course at any bus, either one of them may not be present. The power is transported from one bus to othervia the transmission lines. At any bus *i*, the complex power Si (injected), shown in figure 1, is defined as

$$S_i = S_{Gi} - S_{Di} \qquad (2)$$

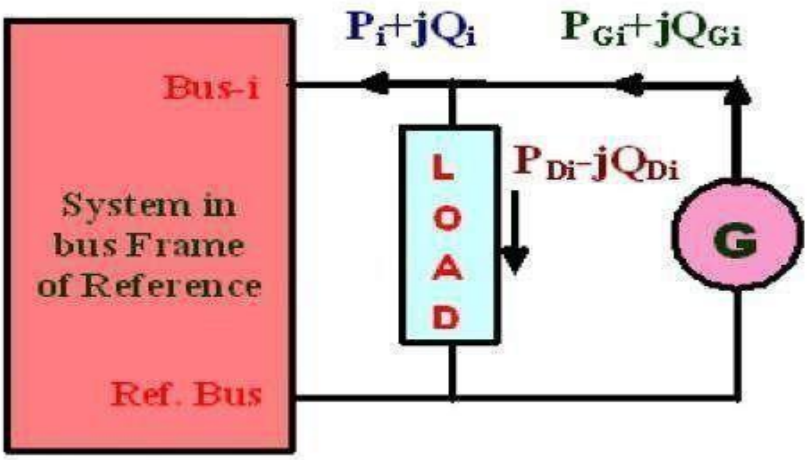


Fig.1 power flows at a bus-i

where Si = net complex power injected into bus *i*, SGi = complex power injected by the generator at bus *i*, and SDi = complex power drawn by the load at bus *i*. According to conservation of complex power, at any bus i, the complex power injected into the bus must be equal to the sum of complex power flows out of the bus via the transmission lines. Hence,

Si = _Sij " i = 1, 2,................n

(3)

where Sij is the sum over all lines connected to the bus and *n* is the number of buses in the system (excluding the ground). The bus current injected at the bus-i is defined as

Ii = IGi – IDi " i = 1, 2, ………..n **(4)** where IGi is the current injected by the generator at the bus and IDi is the current drawn by the load (demand) at that bus. In the bus frame of reference

IBUS = YBUS VBUS
(5)

where

$$I_{BUS} = \begin{bmatrix} I_1 \\ I_2 \\ . \\ . \\ I_n \end{bmatrix}$$ is the vector of currents injected at the buses,

Y_{BUS} is the bus admittance matrix, and

$$V_{BUS} = \begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ V_n \end{bmatrix}$$ is the vector of complex bus voltages.

Equation (5) can be considered as

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j \qquad \forall\ i = 1, 2, \ldots\ldots\ldots n \qquad \textbf{(6)}$$

The complex power S_i is given by

$$S_i = V_i\, I_i^*$$

$$= V_i \left(\sum_{j=1}^{n} Y_{ij}\, V_j \right)^*$$

$$= V_i \left(\sum_{j=1}^{n} Y_{ij}^* V_j^* \right) \qquad \textbf{(7)}$$

Let $V_i \underline{\underline{\Delta}} |V_i| \angle \delta_i = |V_i| (\cos \delta_i + j \sin \delta_i)$

$$\delta_{ij} = \delta_i - \delta_j$$

$$Y_{ij} = G_{ij} + jB_{ij}$$

Hence from (7), we get,

$$S_i = \sum_{j=1}^{n} |V_i|\,|V_j|\,\left(\cos\delta_{ij} + j\sin\delta_{ij}\right)\left(G_{ij} - jB_{ij}\right) \quad \textbf{(8)}$$

Separating real and imaginary parts in (8) we obtain,

$$P_i = \sum_{j=1}^{n} |V_i|\,|V_j|\left(G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}\right) \quad \textbf{(9)}$$

$$Q_i = \sum_{j=1}^{n} |V_i|\,|V_j|\left(G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij}\right) \quad \textbf{(10)}$$

An alternate form of P_i and Q_i can be obtained by representing Y_{ik} also in polar form

as $$Y_{ij} = |Y_{ij}|\angle\theta_{ij} \quad \textbf{(11)}$$

Again, we get from (7),

$$S_i = |V_i|\angle\delta_i \sum_{j=1}^{n} |Y_{ij}|\angle-\theta_{ij}\,|V_j|\angle-\delta_j \quad \textbf{(12)}$$

The real part of (12) gives P_i.

$$P_i = |V_i| \sum_{j=1}^{n} |Y_{ij}|\,|V_j|\,\cos\left(-\theta_{ij} + \delta_i - \delta_j\right)$$

$$= |V_i| \sum_{j=1}^{n} |Y_{ij}|\,|V_j|\,\cos-(\theta_{ij} - \delta_i + \delta_j) \qquad \text{or}$$

$$P_i = \sum_{j=1}^{n} |V_i|\,|V_j|\,|Y_{ij}|\,\cos(\theta_{ij} - \delta_i + \delta_j) \qquad \forall\ i = 1, 2, \ldots\ldots n, \quad \textbf{(13)}$$

Similarly, Q_i is imaginary part of (12) and is given by

$$Q_i = |V_i| \sum_{j=1}^{n} |Y_{ij}|\,|V_j|\,\sin-(\theta_{ij} - \delta_i + \delta_j) \qquad \text{or}$$

$$Q_i = -\sum_{j=1}^{n} |V_i|\,|V_j|\,|Y_{ij}|\,\sin(\theta_{ij} - \delta_i + \delta_j) \qquad \forall\ i = 1, 2, \ldots\ldots n \quad \textbf{(14)}$$

Equations (9)-(10) and (13)-(14) are the „power flow equations" or the „load flow equations" in two alternative forms, corresponding to the n-bus system, where each bus-*i* is characterized by four variables, Pi, Qi, |Vi|, and di. Thus a total of 4n variables are involved in these equations. The load flow equations can be solved for any 2n unknowns, if the other 2n variables are specified. This establishes the need for classification of buses of the system for load flow analysis into: PV bus, PQ bus, etc.

DATA FOR LOAD FLOW

Irrespective of the method used for the solution, the data required is common for any load flow. All data is normally in pu. The bus admittance matrix is formulated from these data. The various data required are as under:

System data: It includes: number of buses-*n*, number of PV buses, number of loads, number of transmission lines, number of transformers, number of shunt elements, the slack bus number, voltage magnitude of slack bus (angle is generally taken as 0o), tolerance limit, base MVA, and maximum permissible number of iterations.

Generator bus data: For every PV bus *i*, the data required includes the bus number, active power generation PGi, the specified voltage magnitude *i sp V* , , minimum reactive power limit Qi,min, and maximum reactive power limit Qi,max.

Load data: For all loads the data required includes the the bus number, active power demand PDi, and the reactive power demand QDi.

Transmission line data: For every transmission line connected between buses *i* and *k* the data includes the starting bus number *i*, ending bus number *k*,.*r*esistance of the line, reactance of the line and the half line charging admittance.

Transformer data:
For every transformer connected between buses *i* and *k* the data to be given includes: the starting bus number *i*, ending bus number *k*, resistance of the transformer, reactance of the transformer, and the off nominal turns-ratio *a*.

Shunt element data: The data needed for the shunt element includes the bus number where element is connected, and the shunt admittance (Gsh + j Bsh).

GAUSS – SEIDEL (GS) METHOD

The GS method is an iterative algorithm for solving non linear algebraic equations. An initial solution vector is assumed, chosen from past experiences, statistical data or from practical considerations. At every subsequent iteration, the solution is updated till convergence is reached. The GS method applied to power flow problem is as discussed below.

Case (a): Systems with PQ buses only:
Initially assume all buses to be PQ type buses, except the slack bus. This means that (n–1) complex bus voltages have to be determined. For ease of programming, the slack bus is generally numbered as bus-1. PV buses are numbered in sequence and PQ buses are ordered next in sequence. This makes programming easier, compared to random ordering of buses. Consider the expression for the complex power at bus-i, given from (7), as:

$$S_i = V_i \left(\sum_{j=1}^{n} Y_{ij} V_j \right)^*$$

This can be written as

$$S_i^* = V_i^* \left(\sum_{j=1}^{n} Y_{ij} V_j \right) \quad \textbf{(15)}$$

Since $S_i^* = P_i - jQ_i$, we get,

$$\frac{P_i - jQ_i}{V_i^*} = \sum_{j=1}^{n} Y_{ij} V_j$$

So that,

$$\frac{P_i - jQ_i}{V_i^*} = Y_{ii} V_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \quad \textbf{(16)}$$

Rearranging the terms, we get,

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \right] \quad \forall\, i = 2, 3, \ldots\ldots\ldots\ldots n \quad \textbf{(17)}$$

Equation (17) is an implicit equation since the unknown variable, appears on both sides of the equation. Hence, it needs to be solved by an iterative technique. Starting from an initial estimate of all bus voltages, in the RHS of (17) the most recent values of the bus voltages is substituted. One iteration of the method involves computation of all the bus voltages. In Gauss–Seidel method, the value of the updated voltages are used in the computation of subsequent voltages in the same iteration, thus speeding up convergence. Iterations are carried out till the magnitudes of all bus voltages do not change by more than the tolerance value. Thus the algorithm for GS method is as under:

Algorithm for GS method

1. Prepare data for the given system as required.
2. Formulate the bus admittance matrix YBUS. This is generally done by the rule of inspection.
3. Assume initial voltages for all buses, 2,3,…n. In practical power systems, the magnitude of the bus voltages is close to 1.0 p.u. Hence, the complex bus voltages at all (n-1) buses (except slack bus) are taken to be $1.0\angle 0^0$. This is normally refered as the ***flat start*** solution.
4. Update the voltages. In any $(k+1)st$ iteration, from (17) the voltages are given by

$$V_i^{(k+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^{(k)})^*} - \sum_{j=1}^{i-1} Y_{ij} V_j^{(k+1)} - \sum_{j=i+1}^{n} Y_{ij} V_j^{(k)}\right] \quad \forall \text{ i=2,3,...n} \qquad \textbf{(18)}$$

Here note that when computation is carried out for bus-i, updated values are already available for buses 2,3….(i-1) in the current *(k+1)st* iteration. Hence these values are used. For buses *(i+1)…..n,* values from previous, *kth* iteration are used.

$$\left|\Delta V_i^{(k+1)}\right| = \left|V_i^{(k+1)} - V_i^{(k)}\right| < \varepsilon \qquad \forall \text{ i = 2,3,...n} \qquad \textbf{(19)}$$

Where,e is the tolerance value. Generally it is customary to use a value of 0.0001 pu. Compute slack bus power after voltages have converged using (15) [assuming bus 1 is slack bus].

$$S_1^* = P_1 - jQ_1 = V_1^*\left(\sum_{j=1}^{n} Y_{1j} V_j\right) \qquad \textbf{(20)}$$

7. Compute all line flows.
8. The complex power loss in the line is given by Sik + Ski. The total loss in the system is calculated by summing the loss over all the lines.

Case (b): Systems with PV buses also present:

At PV buses, the magnitude of voltage and not the reactive power is specified. Hence it is needed to first make an estimate of Qi to be used in (18). From (15) we have

$$Q_i = -\,\text{Im}\left\{ V_i^* \sum_{j=1}^{n} Y_{ij}\, V_j \right\}$$

Where *Im* stands for the imaginary part. At any *(k+1)*st iteration, at the PV bus-*i*,

$$Q_i^{(k+1)} = -\,\text{Im}\left\{ (V_i^{(k)})^* \sum_{j=1}^{i-1} Y_{ij}\, V_j^{(k+1)} + (V_i^{(k)})^* \sum_{j=i}^{n} Y_{ij}\, V_j^{(k)} \right\} \qquad \textbf{(21)}$$

The steps for i^{th} PV bus are as follows:

1. Compute $Q_i^{(k+1)}$ using (21)

2. Calculate V_i using (18) with $Q_i = Q_i^{(k+1)}$

3. Since $|V_i|$ is specified at the PV bus, the magnitude of V_i obtained in step 2

has to be modified and set to the specified value $|V_{i,sp}|$. Therefore,

$$V_i^{(k+1)} = |V_{i,sp}| \angle \delta_i^{(k+1)} \qquad \textbf{(22)}$$

The voltage computation for PQ buses does not change.

Case (c): Systems with PV buses with reactive power generation limits specified:
In the previous algorithm if the Q limit at the voltage controlled bus is violated during any iteration, i.e (*k* +1) *i Q* computed using (21) is either less than Qi, min or greater than Qi,max, it means that the voltage cannot be maintained at the specified value due to lackof reactive power support. This bus is then treated as a PQ bus in the *(k+1)st* iteration and the voltage is calculated with the value of Qi set as follows:

If $Q_i < Q_{i,min}$ Then $Q_i = Q_{i,min}$.

If $Q_i > Q_{i,max}$ Then $Q_i = Q_{i,max}$. **(23)**

If in the subsequent iteration, if Qi falls within the limits, then the bus can be switched back to PV status.

Acceleration of convergence
It is found that in GS method of load flow, the number of iterations increase with increase in the size of the system. The number of iterations required can be reduced if the correction in voltage at each bus is accelerated, by multiplying with a constant α, called the acceleration factor. In the (k+1)st iteration we can let

$$V_i^{(k+1)}(accelerate\ d) = V_i^{(k)} + \alpha\left(V_i^{(k+1)} - V_i^{(k)}\right) \qquad \textbf{(24)}$$

where α is a real number. When α =1, the value of (*k* +1) is the computed value. If 1<α <2 then the value computed is extrapolated. Generally _ is taken between 1.2 to 1.6, for GS load flow procedure. At PQ buses (pure load buses) if the voltage magnitude violates the limit, it simply means that the specified reactive power demand cannot be supplied, with the voltage maintained within acceptable limits.

Examples on GS load flow analysis:

Example-1: *Obtain the voltage at bus 2 for the simple system shown in Fig 2, using the Gauss–Seidel method, if $V_1 = 1\angle 0^0$ pu.*

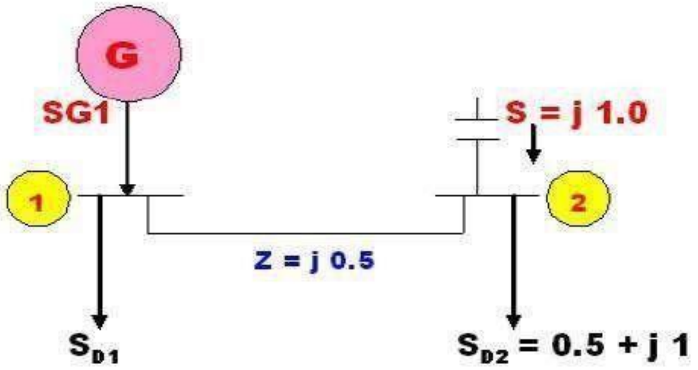


Fig : System of Example 1

Solution:

Here the capacitor at bus 2, injects a reactive power of 1.0 pu. The complex power injection at bus 2 is

$$S_2 = j1.0 - (0.5 + j\,1.0) = -\,0.5 \text{ pu.}$$

$$V_1 = 1\angle 0^0$$

$$Y_{BUS} = \begin{bmatrix} -j2 & j2 \\ j2 & -j2 \end{bmatrix}$$

$$V_2^{(k+1)} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{\left(V_2^{(k)}\right)^*} - Y_{21}\,V_1\right]$$

Since V_1 is specified it is a constant through all the iterations. Let the initial voltage at bus 2, $V_2^0 = 1 + j\,0.0 = 1\angle 0^0$ pu.

$$V_2^1 = \frac{1}{-j2}\left[\frac{-0.5}{1\angle 0^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 1.0 - j0.25 = 1.030776 \ \angle -14.036^0$$

$$V_2^2 = \frac{1}{-j2}\left[\frac{-0.5}{1.030776\angle 14.036^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 0.94118 - j\ 0.23529 \ = 0.970145 \ \angle -14.036^0$$

$$V_2^3 = \frac{1}{-j2}\left[\frac{-0.5}{0.970145\angle 14.036^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 0.9375 - j\ 0.249999 \ = 0.970261 \ \angle -14.931^0$$

$$V_2^4 = \frac{1}{-j2}\left[\frac{-0.5}{0.970261\angle 14.931^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 0.933612 - j\ 0.248963 = 0.966237 \ \angle -14.931^0$$

$$V_2^5 = \frac{1}{-j2}\left[\frac{-0.5}{0.966237\angle 14.931^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 0.933335 - j\ 0.25 \ = 0.966237 \ \angle -14.995^0$$

Since the difference in the voltage magnitudes is less than 10-6 pu, the iterations can be stopped. To compute line flow

$$I_{12} = \frac{V_1 - V_2}{Z_{12}} = \frac{1\angle 0^0 - 0.966237\angle -14.995^0}{j0.5}$$

$$= 0.517472 \angle -14.931^0$$

$$S_{12} = V_1 I_{12}^* = 1\angle 0^0 \times 0.517472 \angle 14.931^0$$

$$= 0.5 + \text{j } 0.133329 \text{ pu}$$

$$I_{21} = \frac{V_2 - V_1}{Z_{12}} = \frac{0.966237\angle -14.995^0 - 1\angle 0^0}{j0.5}$$

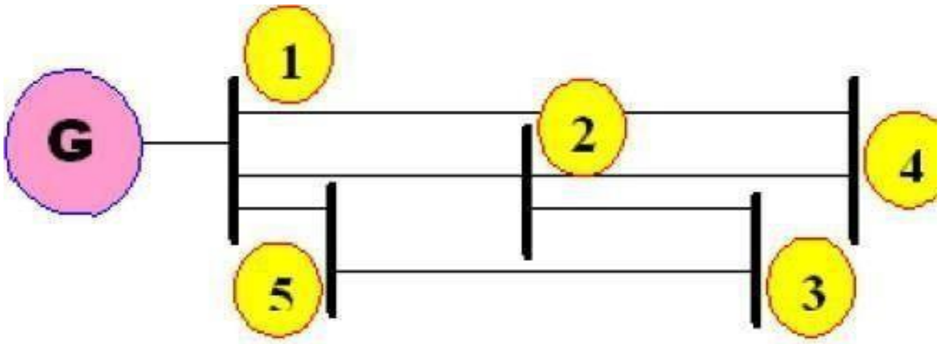
$$= 0.517472 \angle -194.93^0$$

$$S_{21} = V_2 I_{21}^* = -0.5 + \text{j } 0.0 \text{ pu}$$

The total loss in the line is given by S12 + S21 = j 0.133329 pu Obviously, it is observed that there is no real power loss, since the line has no resistance.

Example-2:

For the power system shown in fig. below, with the data as given in tables below, obtain the bus voltages at the end of first iteration, by applying GS method.



Power System of Example 2

Line data of example 2

SB	EB	R (pu)	X (pu)	$\frac{B_C}{2}$
1	2	0.10	0.40	-
1	4	0.15	0.60	-
1	5	0.05	0.20	-
2	3	0.05	0.20	-
2	4	0.10	0.40	-
3	5	0.05	0.20	-

Bus data of example 2

Bus No.	P_G (pu)	Q_G (pu)	P_D (pu)	Q_D (pu)	$\lvert V_{SP}\rvert$ (pu)	δ
1	-	-	-	-	1.02	0°
2	-	-	0.60	0.30	-	-
3	1.0	-	-	-	1.04	-
4	-	-	0.40	0.10	-	-
5	-	-	0.60	0.20	-	-

Solution: In this example, we have,

- Bus 1 is slack bus, Bus 2, 4, 5 are PQ buses, and Bus 3 is PV bus
- The lines do not have half line charging admittances

  $P_2 + jQ_2 = P_{G2} + jQ_{G2} - (P_{D2} + jQ_{D2}) = -0.6 - j0.3$

$P_3 + jQ_3 = P_{G3} + jQ_{G3} - (P_{D3} + jQ_{D3}) = 1.0 + jQ_{G3}$

Similarly $P_4 + jQ_4 = -0.4 - j0.1$, $P_5 + jQ_5 = -0.6 - j0.2$

The Y_{bus} formed by the rule of inspection is given by:

$Y_{bus} =$

2.15685 -j8.62744	-0.58823 +j2.35294	0.0+j0.0	-0.39215 +j1.56862	-1.17647 +j4.70588
-0.58823 +j2.35294	2.35293 -j9.41176	-1.17647 +j4.70588	-0.58823 +j2.35294	0.0+j0.0
0.0+j0.0	-1.17647 +j4.70588	2.35294 -j9.41176	0.0+j0.0	-1.17647 +j4.70588
-0.39215 +j1.56862	-0.58823 +j2.35294	0.0+j0.0	0.98038 -j3.92156	0.0+j0.0
-1.17647 +j4.70588	0.0+j0.0	-1.17647 +j4.70588	0.0+j0.0	2.35294 -j9.41176

The voltages at all PQ buses are assumed to be equal to 1+j0.0 pu. The slack bus voltage is taken to be V_1^0 = 1.02+j0.0 in all iterations.

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{o*}} - Y_{21}V_1^o - Y_{23}V_3^0 - Y_{24}V_4^0 - Y_{25}V_5^0\right]$$

$$= \frac{1}{Y_{22}}\left[\frac{-0.6 + j0.3}{1.0 - j0.0} - \{(-0.58823 + j2.35294)\times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-1.17647 + j4.70588)\times 1.04\angle 0^o\} - \{(-0.58823 + j2.35294)\times 1.0\angle 0^o\}\right]$$

$$= 0.98140\angle -3.0665^o = 0.97999 - j0.0525$$

Bus 3 is a PV bus. Hence, we must first calculate Q_3. This can be done as under:

$$Q_3 = |V_3||V_1|(G_{31}\sin\delta_{31} - B_{31}\cos\delta_{31}) + |V_3||V_2|(G_{32}\sin\delta_{32} - B_{32}\cos\delta_{32})$$

$$+ |V_3|^2(G_{33}\sin\delta_{33} - B_{33}\cos\delta_{33}) + |V_3||V_4|(G_{34}\sin\delta_{34} - B_{34}\cos\delta_{34})$$

$$+ |V_3||V_5|(G_{35}\sin\delta_{35} - B_{35}\cos\delta_{35})$$

We note that $\delta_1 = 0^o$; $\delta_2 = -3.0665^o$; $\delta_3 = 0^o$; $\delta_4 = 0^o$ and $\delta_5 = 0^o$

$\therefore \delta_{31} = \delta_{33} = \delta_{34} = \delta_{35} = 0^o$ ($\delta_{ik} = \delta_i - \delta_k$); $\delta_{32} = 3.0665^o$

$Q_3 = 1.04\,[1.02\,(0.0+j0.0) + 0.9814\,\{-1.17647\times\sin(3.0665^o) - 4.70588$
$\times\cos(3.0665^o)\} + 1.04\{-9.41176\times\cos(0^o)\} + 1.0\,\{0.0 + j0.0\} + 1.0\{-4.70588\times\cos(0^0)\}]$

$= 1.04\,[-4.6735 + 9.78823 - 4.70588] = 0.425204$ pu.

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{o*}} - Y_{31}V_1^o - Y_{32}V_2^1 - Y_{34}V_4^0 - Y_{35}V_5^0\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{1.0 - j0.425204}{1.04 - j0.0} - \{(-1.7647 + j4.70588)\times(0.98140\angle -3.0665^o)\}\right.$$

$$\left. - \{(-1.17647 + j4.70588)\times(1\angle 0^o)\}\right]$$

$$= 1.05569\angle 3.077^o = 1.0541 + j0.05666 \text{ pu.}$$

Since it is a PV bus, the voltage magnitude is adjusted to specified value and V_3^1 is computed as: $V_3^1 = 1.04\angle 3.077^0$ pu

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{o*}} - Y_{41}V_1^o - Y_{42}V_2^1 - Y_{43}V_3^1 - Y_{45}V_5^0\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{-0.4 + j0.1}{1.0 - j0.0} - \{(-0.39215 + j1.56862)\times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-0.58823 + j2.35294)\times(0.98140\angle -3.0665^o)\}\right]$$

$$= \frac{0.45293 - j3.8366}{0.98038 - j3.92156} = 0.955715\angle -7.303^o \text{ pu} = 0.94796 - j0.12149$$

$$V_5^1 = \frac{1}{Y_{55}}\left[\frac{P_5 - jQ_5}{V_5^{o*}} - Y_{51}V_1^o - Y_{52}V_2^1 - Y_{53}V_3^1 - Y_{54}V_4^1\right]$$

$$= \frac{1}{Y_{55}}\left[\frac{-0.6 + j0.2}{1.0 - j0.0} - \{(-1.17647 + j4.70588)\times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-1.17647 + j4.70588)\times 1.04\angle 3.077^o\}\right]$$

$$= 0.994618\angle -1.56^o = 0.994249 - j0.027$$

Thus at end of 1st iteration, we have,

$V_1 = 1.02\angle 0^0$ pu $\quad V_2 = 0.98140\angle -3.066^0$ pu

$V_3 = 1.04\angle 3.077^0$ pu $\quad V_4 = 0.955715\angle -7.303^0$ pu

and $\quad V_5 = 0.994618\angle -1.56^0$ pu

Example-3:

Obtain the load flow solution at the end of first iteration of the system with data as given below. The solution is to be obtained for the following cases

(i) All buses except bus 1 are PQ Buses

(ii) Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu

(iii) Bus 2 is PV bus, with voltage magnitude specified as 1.04 and 0.25_Q2_1.0 pu.

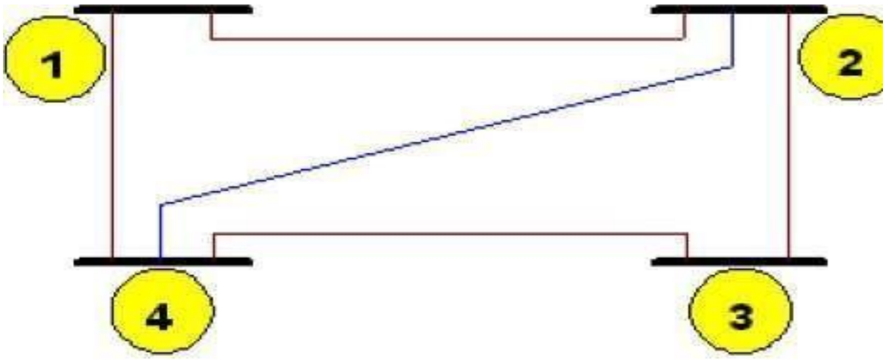


Fig. System for Example 3

Table: Line data of example 3

SB	EB	R (pu)	X (pu)
1	2	0.05	0.15
1	3	0.10	0.30
2	3	0.15	0.45
2	4	0.10	0.30
3	4	0.05	0.15

Table: Bus data of example 3

Bus No.	P_i (pu)	Q_i (pu)	V_i
1	–	–	$1.04\angle 0^0$
2	0.5	– 0.2	–
3	– 1.0	0.5	–
4	– 0.3	– 0.1	–

Solution: Note that the data is directly in terms of injected powers at the buses. The bus admittance matrix is formed by inspection as under:

$Y_{BUS} =$

3.0 – j9.0	–2.0 + j6.0	– 1.0 + j3.0	0
–2.0 + j6.0	3.666 – j11.0	– 0.666 + j2.0	– 1.0 + j3.0
–1.0 + j3.0	–0.666 + j2.0	3.666 – j11.0	–2.0 + j6.0
0	–1.0 + j3.0	– 2.0 + j6.0	3.0 – j9.0

Case(i): All buses except bus 1 are PQ Buses

Assume all initial voltages to be $1.0 \angle 0^0$ pu.

$$V_2^1 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{V_2^{o*}} - Y_{21} V_1^o - Y_{23} V_3^0 - Y_{24} V_4^0 \right]$$

$$= \frac{1}{Y_{22}}\left[\frac{0.5 + j0.2}{1.0 - j0.0} - \{(-2.0 + j6.0)\times(1.04\angle 0^o)\}\right.$$

$$\left. - \{(-0.666 + j2.0)\times(1.0\angle 0^o)\} - \{(-1.0 + j3.0)\times(1.0\angle 0^0)\}\right]$$

$$= 1.02014 \angle 2.605^o$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{o*}} - Y_{31}\, V_1^o - Y_{32}\, V_2^1 - Y_{34}\, V_4^0\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{-1.0 - j0.5}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.04\angle 0.0^o)\}\right.$$

$$\left. - \{(-0.666 + j2.0)\times(1.02014\angle 2.605^o)\} - \{(-2.0 + j6.0)\times(1.0\angle 0^0)\}\right]$$

$$= 1.03108 \angle -4.831^o$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{o*}} - Y_{41}\, V_1^o - Y_{42}\, V_2^1 - Y_{43}\, V_3^1\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{0.3 + j0.1}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.02014\angle 2.605^o)\}\right.$$

$$\left. - \{(-2.0 + j6.0)\times(1.03108\angle -4.831^o)\}\right]$$

$$= 1.02467 \angle -0.51^o$$

Hence

$V_1^1 = 1.04 \angle 0^0$ pu $\qquad V_2^1 = 1.02014 \angle 2.605^0$ pu

$V_3^1 = 1.03108 \angle -4.831^0$ pu $\qquad V_4^1 = 1.02467 \angle -0.51^0$ pu

Case(ii): Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu

We first compute Q_2.

$$Q_2 = |V_2|\Big[|V_1|\left(G_{21}\sin\delta_{21} - B_{21}\cos\delta_{21}\right) + |V_2|\left(G_{22}\sin\delta_{22} - B_{22}\cos\delta_{22}\right) + |V_3|\left(G_{23}\sin\delta_{23} - B_{23}\cos\delta_{23}\right) + |V_4|\left(G_{24}\sin\delta_{24} - B_{24}\cos\delta_{24}\right)\Big]$$

$$= 1.04\ [1.04\ \{-6.0\} + 1.04\ \{11.0\} + 1.0\{-2.0\} + 1.0\ \{-3.0\} = 0.208 \text{ pu.}$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{0.5 - j0.208}{1.04\angle 0^0} - \{(-2.0 + j6.0)\times(1.04\angle 0^o)\} - \{(-0.666 + j2.0)\times(1.0\angle 0^o)\} - \{(-1.0 + j3.0)\times(1.0\angle 0^o)\}\right]$$

$$= 1.051288 + j0.033883$$

The voltage magnitude is adjusted to 1.04. Hence $V_2^1 = 1.04\angle 1.846^0$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{-1.0 - j0.5}{1.0\ \angle 0^0} - \{(-1.0 + j3.0)\times(1.04\angle 0.0^o)\} - \{(-0.666 + j2.0)\times(1.04\angle 1.846^o)\} - \{(-2.0 + j6.0)\times(1.0\angle 0^0)\}\right]$$

$$= 1.035587\angle -4.951^o \text{ pu.}$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{0.3 + j0.1}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.04\angle 1.846^o)\} - \{(-2.0 + j6.0)\times(1.035587\angle -4.951^o)\}\right]$$

$$= 0.9985\angle -0.178^o$$

Hence at end of 1st iteration we have:

$V_1^1 = 1.04\ \angle 0^0$ pu $\qquad V_2^1 = 1.04\ \angle 1.846^0$ pu

$V_3^1 = 1.035587\ \angle -4.951^0$ pu $\qquad V_4^1 = 0.9985\angle -0.178^0$ pu

Case (iii): Bus 2 is PV bus, with voltage magnitude specified as 1.04 & $0.25 \le Q_2 \le 1$ pu. If $0.25 \le Q_2 \le 1.0$ pu then the computed value of $Q_2 = 0.208$ is less than the lower limit. Hence, Q_2 is set equal to 0.25 pu. Iterations are carried out with this value of Q_2. The voltage magnitude at bus 2 can no longer be maintained at 1.04. Hence, there is no necessity to adjust for the voltage magnitude. Proceeding as before we obtain at the end of first iteration,

$$V_1^1 = 1.04 \angle 0^0 \text{ pu} \qquad V_2^1 = 1.05645 \angle 1.849^0 \text{ pu}$$

$$V_3^1 = 1.038546 \angle -4.933^0 \text{ pu} \qquad V_4^1 = 1.081446 \angle 4.896^0 \text{ pu}$$

Limitations of GS load flow analysis

GS method is very useful for very small systems. It is easily adoptable, it can be generalized and it is very efficient for systems having less number of buses. However, GS LFA fails to converge in systems with one or more of the features as under:

- Systems having large number of radial lines
- Systems with short and long lines terminating on the same bus
- Systems having negative values of transfer admittances
- Systems with heavily loaded lines, etc.

GS method successfully converges in the absence of the above problems. However, convergence also depends on various other set of factors such as: selection of slack bus, initial solution, acceleration factor, tolerance limit, level of accuracy of results needed, type and quality of computer/ software used, etc.

NEWTON –RAPHSON METHOD

Newton-Raphson (NR) method is used to solve a system of non-linear algebraic equations of the form f(x) =0. Consider a set of *n* non-linear algebraic equations given by

$$f_i(x_1, x_2 ...... x_n) = 0 \qquad i = 1,2......n \qquad \textbf{(25)}$$

Let $x_1^0, x_2^0 .... x_n^0$, be the initial guess of unknown variables and $\Delta x_1^0, \Delta x_2^0 .... \Delta x_n^0$ be the respective corrections. Therefore,

$$f_i(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0 .... x_n^0 + \Delta x_n^0) = 0 \qquad i = 1,2......n \qquad \textbf{(26)}$$

The above equation can be expanded using Taylor's series to give

$$f_i(x_1^0, x_2^0 .... x_n^0) + \left[\left(\frac{\partial f_i}{\partial x_1} \right)^0 \Delta x_1^0 + \left(\frac{\partial f_i}{\partial x_2} \right) \Delta x_2^0 + ... + \left(\frac{\partial f_i}{\partial x_n} \right) \Delta x_n^0 \right]$$

$$+ \text{Higher order terms} = 0 \qquad \forall \; i = 1,2......n \qquad \textbf{(27)}$$

Where, $\left(\frac{\partial f_i}{\partial x_1} \right)^0, \left(\frac{\partial f_i}{\partial x_2} \right)^0, ... \left(\frac{\partial f_i}{\partial x_n} \right)^0$ are the partial derivatives of f_i with respect to $x_1, x_2 .... x_n$ respectively, evaluated at ($x_1^0, x_2^0 .... x_0^n$). If the higher order terms are neglected, then (27) can be written in matrix form as

$$\begin{bmatrix} f_1^0 \\ f_2^0 \\ . \\ . \\ . \\ f_n^0 \end{bmatrix} + \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1} \right)^0 & \left(\frac{\partial f_1}{\partial x_2} \right)^0 & . \; . \; . & \left(\frac{\partial f_1}{\partial x_n} \right)^0 \\ \left(\frac{\partial f_2}{\partial x_1} \right)^0 & \left(\frac{\partial f_2}{\partial x_2} \right)^0 & . \; . \; . & \left(\frac{\partial f_2}{\partial x_n} \right)^0 \\ . & . & . \; . \; . & . \\ . & . & . \; . \; . & . \\ . & . & . \; . \; . & . \\ \left(\frac{\partial f_n}{\partial x_1} \right) & \left(\frac{\partial f_n}{\partial x_2} \right) & . \; . \; . & \left(\frac{\partial f_n}{\partial x_n} \right) \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ . \\ . \\ . \\ \Delta x_n^0 \end{bmatrix} = 0 \qquad \textbf{(28)}$$

In vector form (28) can be written as

$$F^0 + J^0 \Delta X^0 = 0$$

Or $$F^0 = -J^0 \Delta X^0$$

Or $$\Delta X^0 = -[J^0]^{-1} F^0 \qquad \textbf{(29)}$$

And $$X^1 = X^0 + \Delta X^0 \qquad \textbf{(30)}$$

Here, the matrix [J] is called the **Jacobian** matrix. The vector of unknown variables is updated using (30). The process is continued till the difference between two successive iterations is less than the tolerance value.

NR method for load flow solution in polar coordinates

In application of the NR method, we have to first bring the equations to be solved, to the form $f_i(x_1, x_2, ... x_n) = 0$, where $x_1, x_2, ... x_n$ are the unknown variables to be determined. Let us assume that the power system has n_1 PV buses and n_2 PQ buses. In polar coordinates the unknown variables to be determined are:

(i) δ_i, the angle of the complex bus voltage at bus *i*, at all the PV and PQ buses. This gives us $n_1 + n_2$ unknown variables to be determined.

(ii) $|V_i|$, the voltage magnitude of bus *i*, at all the PQ buses. This gives us n_2 unknown variables to be determined.

Therefore, the total number of unknown variables to be computed is: $n_1 + 2n_2$, for which we need $n_1 + 2n_2$ consistent equations to be solved. The equations are given by,

$$\Delta P_i = P_{i,sp} - P_{i,cal} = 0 \qquad \textbf{(31)}$$

$$\Delta Q_i = Q_{i,sp} - Q_{i,cal} = 0 \qquad \textbf{(32)}$$

Where $P_{i,sp}$ = Specified active power at bus *i*

$Q_{i,sp}$ = Specified reactive power at bus *i*

$P_{i,cal}$ = Calculated value of active power using voltage estimates.

$Q_{i,cal}$ = Calculated value of reactive power using voltage estimates

ΔP = Active power residue

ΔQ = Reactive power residue

The real power is specified at all the PV and PQ buses. Hence (31) is to be solved at all PV and PQ buses leading to $n_1 + n_2$ equations. Similarly the reactive power is specified at all the PQ buses. Hence, (32) is to be solved at all PQ buses leading to n_2 equations.

We thus have $n_1 + 2n_2$ equations to be solved for $n_1 + 2n_2$ unknowns. (31) and (32) are of the form F(x) = 0. Thus NR method can be applied to solve them. Equations (31) and (32) can be written in the form of (30) as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \quad \textbf{(33)}$$

Where J_1, J_2, J_3, J_4 are the negated partial derivatives of ΔP and ΔQ with respect to corresponding δ and $|V|$. The negated partial derivative of ΔP, is same as the partial derivative of P_{cal}, since P_{sp} is a constant. The various computations involved are discussed in detail next.

Computation of P_{cal} and Q_{cal}:

The real and reactive powers can be computed from the load flow equations as:

$$P_{i,Cal} = P_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

$$= G_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) \quad \textbf{(34)}$$

$$Q_{i,Cal} = Q_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

$$= -B_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}) \quad \textbf{(35)}$$

The powers are computed at any $(r+1)^{st}$ iteration by using the voltages available from previous iteration. The elements of the Jacobian are found using the above equations as:

Elements of J_1

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|\{G_{ik}(-\sin\delta_{ik}) + B_{ik}\cos\delta_{ik}\}$$

$$= -Q_i - B_{ii}|V_i|^2$$

$$\frac{\partial P_i}{\partial \delta_k} = |V_i||V_k|(G_{ik}(-\sin\delta_{ik})(-1) + B_{ik}(\cos\delta_{ik})(-1))$$

Elements of J_3

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_i||V_k|(G_{ik} \cos\delta_{ik} + B_{ik} \sin\delta_{ik}) = P_i - G_{ii}|V_i|^2$$

$$\frac{\partial Q_i}{\partial \delta_k} = -|V_i||V_k|(G_{ik} \cos\delta_{ik} + B_{ik} \sin\delta_{ik})$$

Elements of J_2

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = 2|V_i|^2 G_{ii} + |V_i|\sum_{\substack{k=1 \\ k \neq i}}^{n} |V_k|(G_{ik} \cos\delta_{ik} + B_{ik} \sin\delta_{ik}) = P_i + |V_i|^2 G$$

$$\frac{\partial P_i}{\partial |V_k|}|V_k| = |V_i||V_k|(G_{ik} \cos\delta_{ik} + B_{ik} \sin\delta_{ik})$$

Elements of J_4

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = -2|V_i|^2 B_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_i||V_k|(G_{ik} \sin\delta_{ik} - B_{ik} \cos\delta_{ik}) = Q_i - |V_i|^2$$

$$\frac{\partial Q_i}{\partial |V_k|}|V_k| = |V_i||V_k|(G_{ik} \sin\delta_{ik} - B_{ik} \cos\delta_{ik})$$

Thus, the linearized form of the equation could be considered agai

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \dfrac{\Delta|V|}{|V|} \end{bmatrix}$$

The elements are summarized below:

(i) $H_{ii} = \dfrac{\partial P_i}{\partial \delta_i} = -Q_i - B_{ii}|V_i|^2$

(ii) $H_{ik} = \dfrac{\partial P_i}{\partial \delta_k} = a_k f_i - b_k e_i = |V_i||V_k|(G_{ik} \sin\delta_{ik} - B_{ik} \cos\delta_{ik})$

(iii) $N_{ii} = \dfrac{\partial P_i}{\partial |V_i|}|V_i| = P_i + G_{ii}|V_i|^2$

(iv) $N_{ik} = \dfrac{\partial P_i}{\partial |V_k|}|V_k| = a_k e_i + b_k f_i = |V_i||V_k|(G_{ik} \cos\delta_{ik} + B_{ik} \sin\delta_{ik})$

(v) $M_{ii} = \dfrac{\partial Q_i}{\partial \delta_i} = P_i - G_{ii}|V_i|^2$